Oct. 2024

5th International Symposium on Probabilistic Methodologies for Nuclear Applications

UC 02

Stress intensity factor solutions for circumferential and axial semi-elliptical surface cracks with large aspect ratios in pipes

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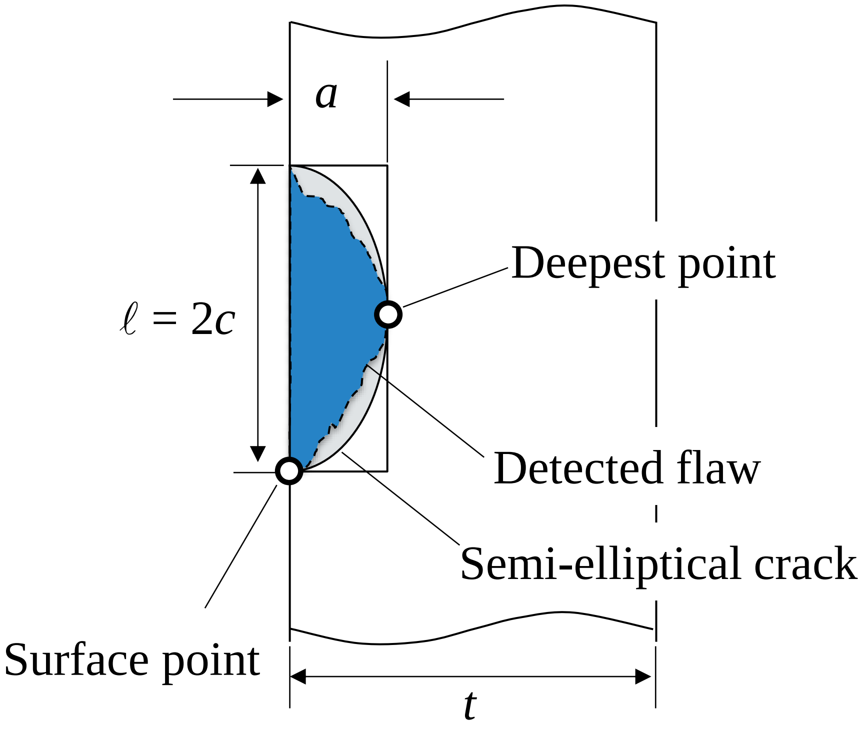
Introduction

- Structural integrity analysis is essential to assessing whether a component can safely withstand the service conditions throughout its operation time.
- A surface flaw detected by non-destructive testing is often modeled as a semi-elliptical crack in various codes and standards.



Stress intensity factor solutions

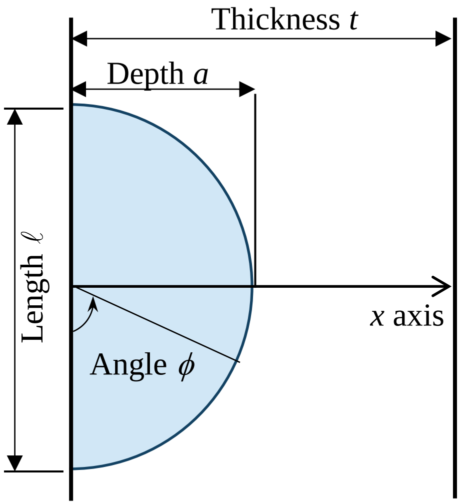
- Crack growth rates at the surface and deepest points can be predicted by the stress intensity factor K_I.
- The stress intensity factor K_I depends on the applied stress and geometrical parameters.

Method of K_I determination (ASME Section XI, A-3000)*

Applied stresses normal to the plane are represented by a polynomial relation.

$$\sigma(x) = A_0 + A_1\left(\frac{x}{a}\right) + A_2\left(\frac{x}{a}\right)^2 + A_3\left(\frac{x}{a}\right)^3 + A_4\left(\frac{x}{a}\right)^4$$

A_i : Stress coefficients ($\mathrm{i} = 0 - 4$)



The stress intensity factor K_I is given by

$$K_I = \left[(A_0 + A_p)G_0 + A_1G_1 + A_2G_2 + A_3G_3 + A_4G_4\right]\sqrt{\frac{\pi a}{Q}}$$

Geometrical parameters
$$\begin{cases} Q : \text{flaw shape parameter} \\ G_\mathrm{i} : \text{influence coefficients } (\mathrm{i} = 0 - 4) \end{cases}$$

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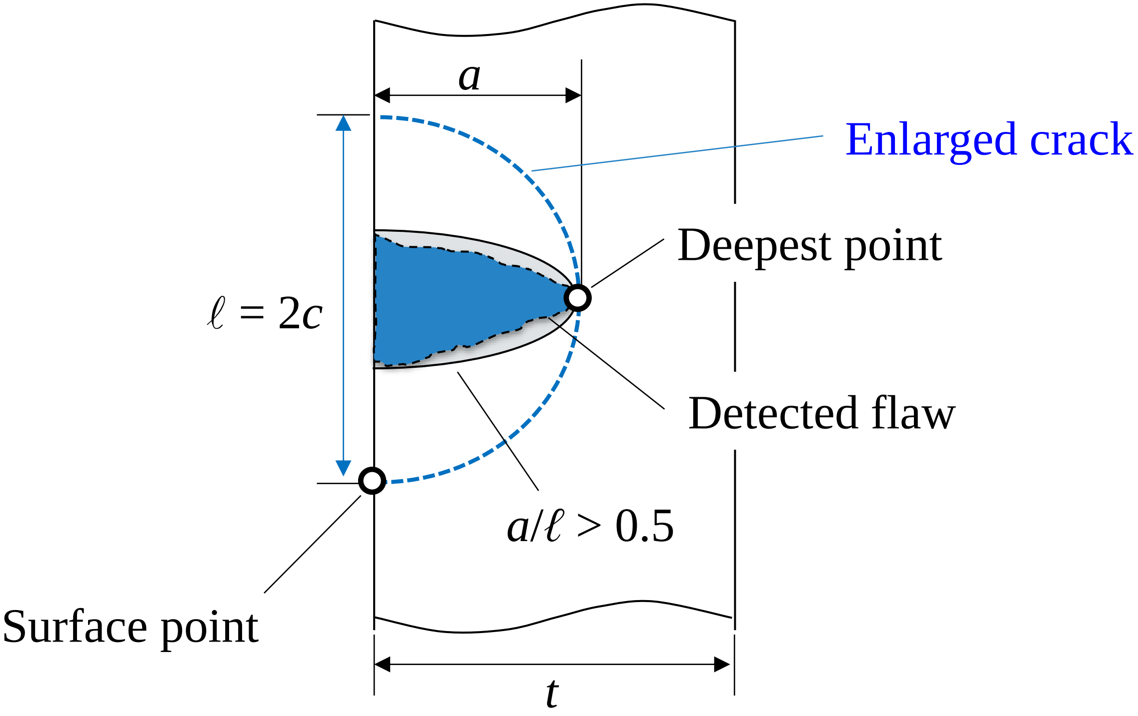
* ASME boiler and pressure vessel code Section XI, Rules for Inservice Inspection of Nuclear Power Plant Components, 2023.

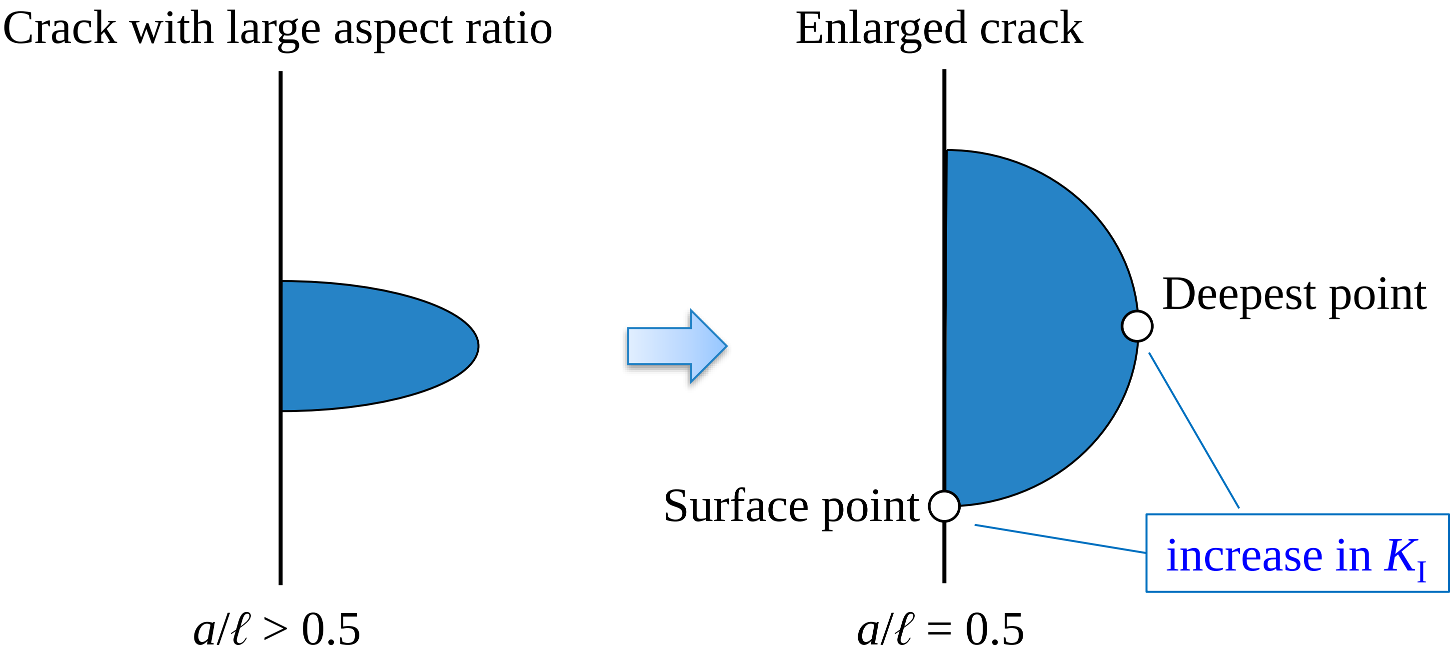
Crack with large aspect ratio ($a/\ell > 0.5$)

- Stress corrosion cracking (SCC) often produces deep cracks whose depth a can be deeper than its half-crack length c (cracks with large aspect ratios).
- By the ASME Code Section XI, a deep surface crack of this type is conservatively modeled as a larger semi-circular crack with the same depth ($a/\ell = 0.5$).



Objectives

Although enlarging crack size seems to be conservative for flaw evaluation, the rate of increase in K_I (or G_i) is unclear in this procedure.



Crack with large aspect ratio

Enlarged crack

Deepest point

Surface point

increase in K_I

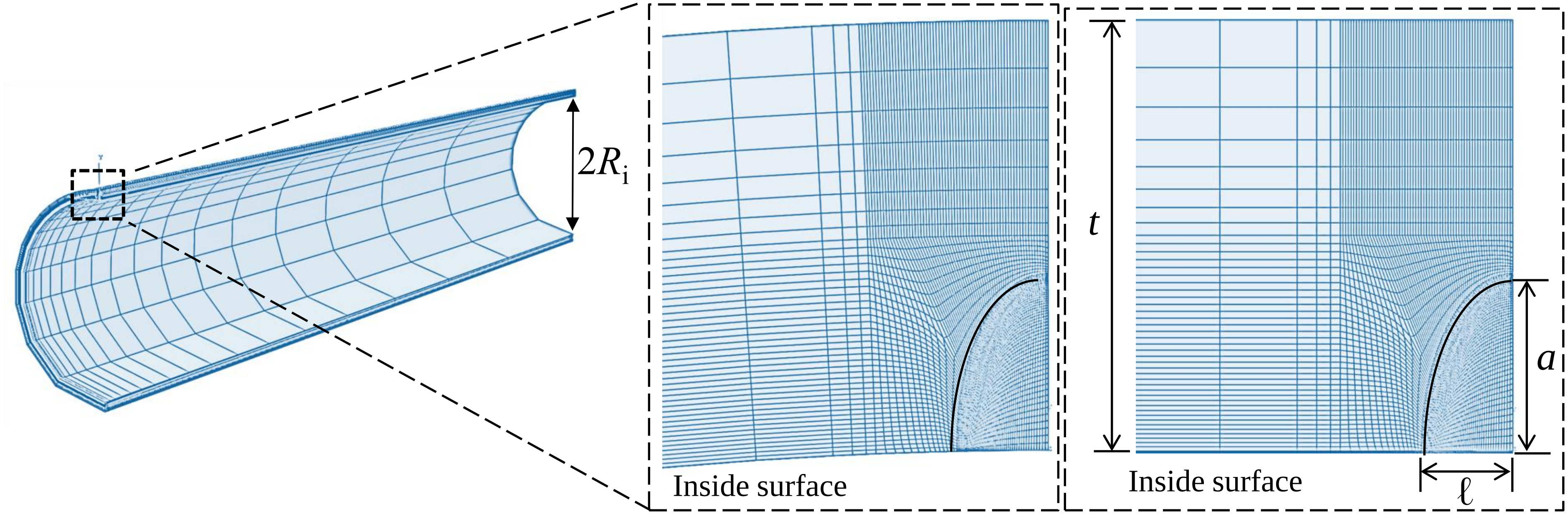
$a/\ell > 0.5$

$a/\ell = 0.5$

The objective of this study is to investigate how enlarging crack size affects the stress intensity factor for a crack with a large aspect ratio.

Finite element models

We referred to finite element K_I solutions for axial and circumferential cracks with $a/\ell > 0.5$ developed by Li et al.*



Cylindrical model

Circumferential crack

Axial crack

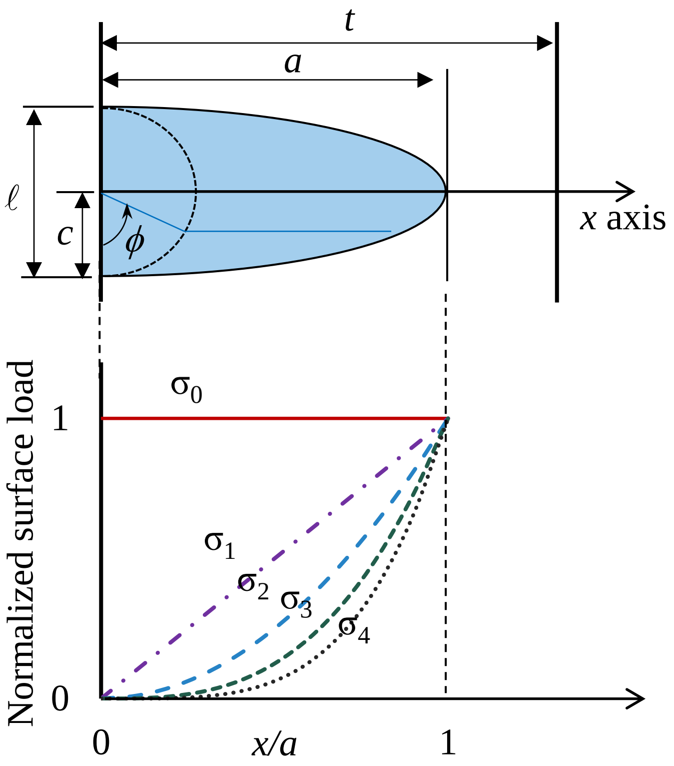
Parameters for FE models

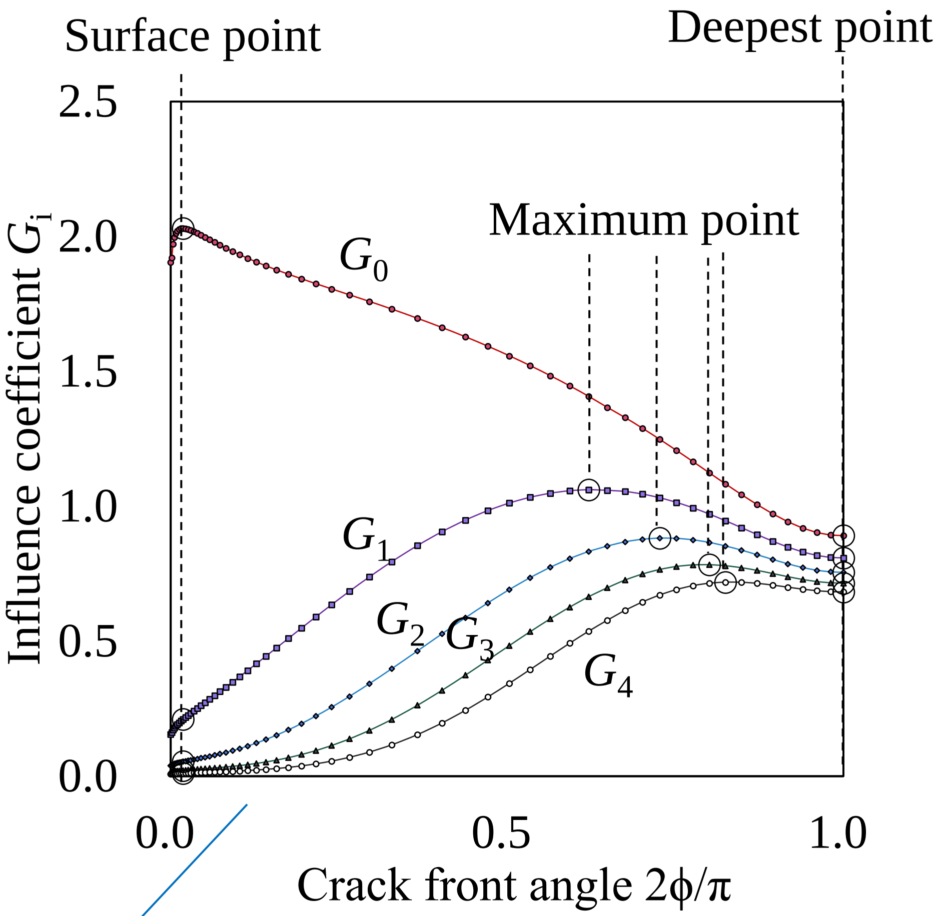
Crack aspect ratio	a/ℓ	0.5, 1.0, 2.0, 4.0
Crack depth ratio	a/t	0.01, 0.1, 0.2, 0.4, 0.6, 0.8
Inside radius to thickness of cylinder	R_i/t	2, 5, 10, 20, 40, 80

* Li, Y., et al, ASME Journal of Pressure Vessel Technology, 2017; 139(2): 021202.

FEA solutions (G_i values)

Semi-elliptical crack with $a/\ell = 2.0$

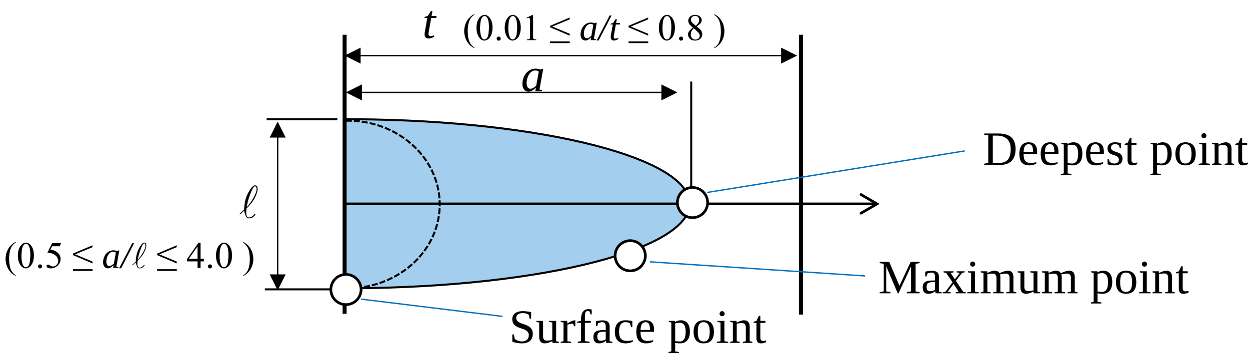




Care should be taken when $a/\ell > 0.5$: G_1–G_4 values might be highest at an intermediate point (maximum point) rather than at the surface or deepest point.

Closed-form solutions

All the FEA solutions at surface, maximum, and deepest points were fitted to the following equation.



Closed-form G_i solutions*

$$G_i = k_{00} + k_{10}\left(\frac{a}{t}\right) + k_{01}\left(\frac{a}{\ell}\right) + k_{20}\left(\frac{a}{t}\right)^2 + k_{02}\left(\frac{a}{\ell}\right)^2 + k_{11}\left(\frac{a}{t}\right)\left(\frac{a}{\ell}\right)$$
$$+ k_{30}\left(\frac{a}{t}\right)^3 + k_{03}\left(\frac{a}{\ell}\right)^3 + k_{12}\left(\frac{a}{t}\right)\left(\frac{a}{\ell}\right)^2 + k_{21}\left(\frac{a}{t}\right)^2\left(\frac{a}{\ell}\right)$$

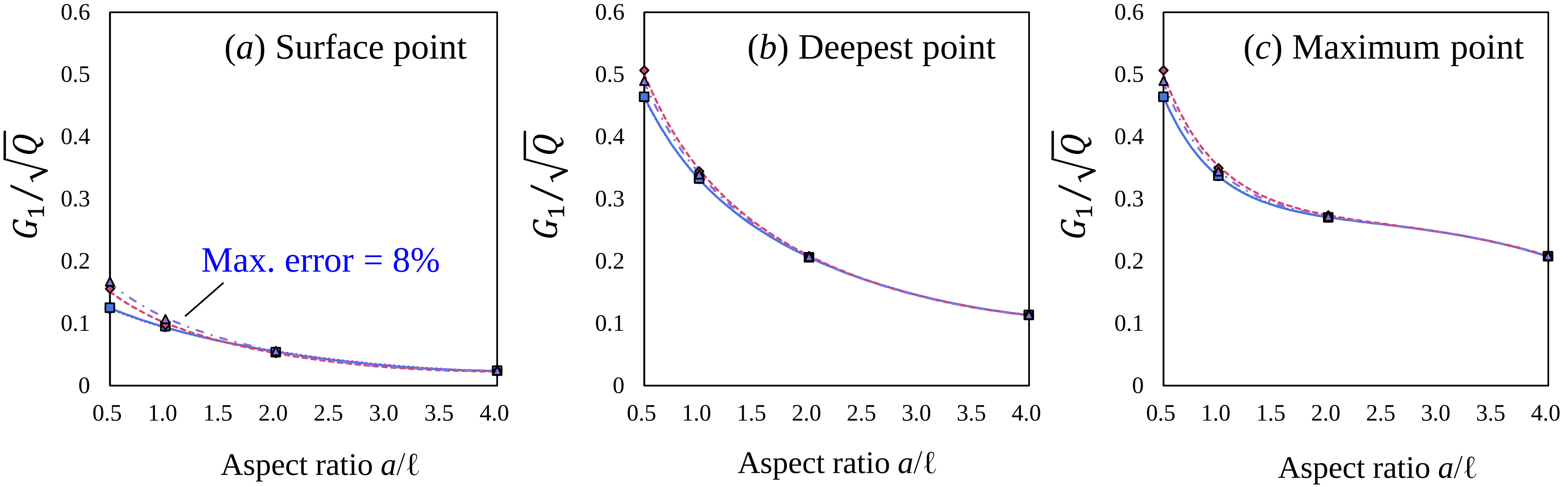
G_i : Influence coefficients k_{ij} : Fitting parameter

The fitting results of axial cracks are demonstrated in the following.

* Azuma, K., et al., ASME. J. Pressure Vessel Technol. December 2022; 144(6): 061303.

Curve fitting for G_i solutions

G_1 solutions for an axial crack



Small crack in thick pipe

········	Fitting	$a/t = 0.01$ $R/t = 2$
●	FEA	

Large crack in thick pipe

- - - -	Fitting	$a/t = 0.8$ $R/t = 2$
◆	FEA	

Small crack in thin pipe

——	Fitting	$a/t = 0.01$ $R/t = 80$
■	FEA	

Large crack in thin pipe

- · -	Fitting	$a/t = 0.8$ $R/t = 80$
▲	FEA	

- The maximum relative fitting error is 8% for the surface point.
- The effect of *a/ℓ* is the most significant among the geometric parameters.

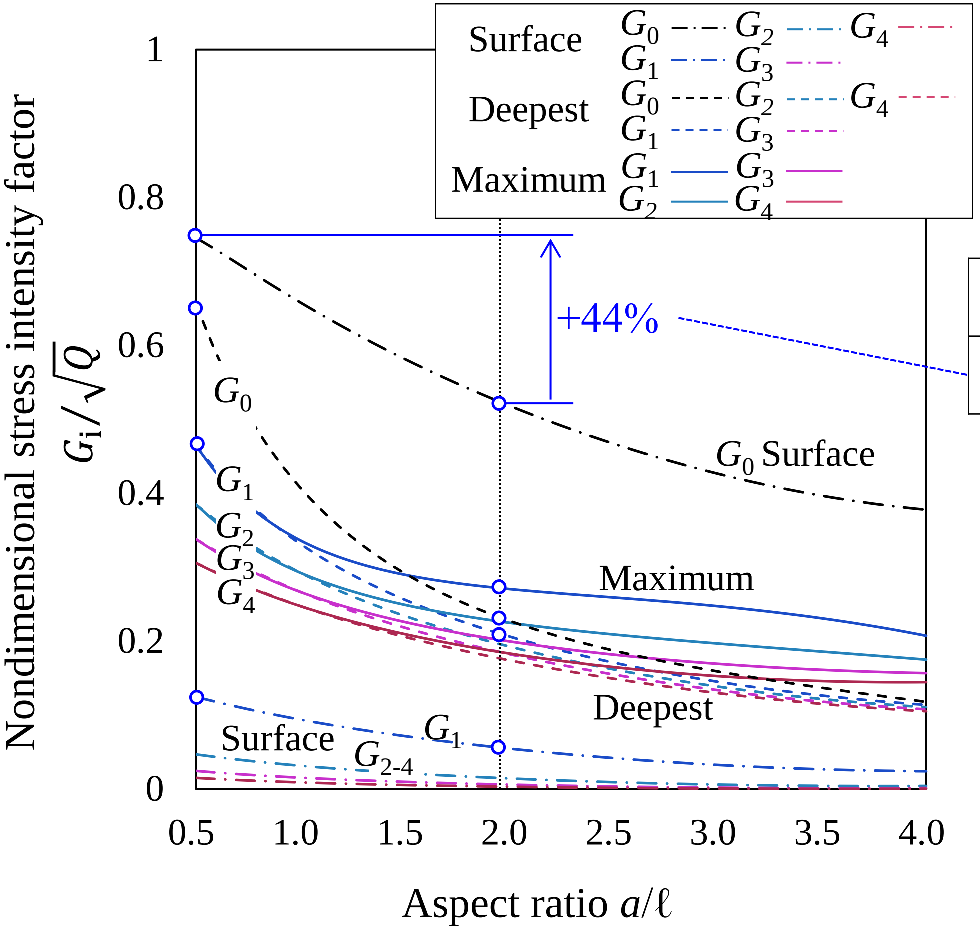
Fitting parameters

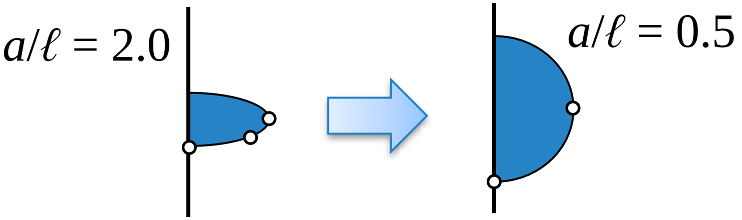
Fitting coefficients for the closed-form solution at the surface point for an axial crack

	R_i/t	k_{00}	k_{10}	k_{01}	k_{20}	k_{02}	k_{11}	k_{30}	k_{03}	k_{12}	k_{21}
G_0	**2**	0.6617	-0.0830	1.1449	0.5526	-0.2836	-0.2494	-0.0603	0.0293	0.0568	-0.1003
	5	0.6516	0.0514	1.1727	0.4506	-0.3020	-0.2307	-0.0464	0.0323	0.0527	-0.1024
	10	0.6429	0.0903	1.1921	0.4776	-0.3126	-0.2355	-0.0813	0.0337	0.0538	-0.1048
	20	0.6524	0.1352	1.1665	0.4154	-0.3001	-0.2333	-0.0451	0.0321	0.0509	-0.0958
	40	0.6629	0.1361	1.1408	0.4234	-0.2871	-0.2281	-0.0341	0.0303	0.0503	-0.1018
	80	0.6491	0.1437	1.1928	0.4040	-0.3203	-0.2373	-0.0251	0.0355	0.0519	-0.0970
G_1	**2**	0.1407	-0.0166	0.1377	0.1569	-0.0659	-0.0597	-0.0264	0.0081	0.0151	-0.0309
	5	0.1294	0.0562	0.1625	0.1012	-0.0807	-0.0826	-0.0503	0.0105	0.0144	-0.0021
	10	0.1375	0.0416	0.1457	0.1475	-0.0709	-0.0795	-0.0747	0.0090	0.0147	-0.0093
	20	0.1392	0.0456	0.1415	0.1499	-0.0692	-0.0788	-0.0766	0.0088	0.0143	-0.0087
	40	0.1431	0.0512	0.1314	0.1468	-0.0639	-0.0798	-0.0750	0.0080	0.0143	-0.0082
	80	0.1388	0.0522	0.1475	0.1407	-0.0742	-0.0820	-0.0707	0.0097	0.0147	-0.0071
G_2	**2**	0.0616	-0.0072	0.0335	0.0872	-0.0250	-0.0281	-0.0260	0.0036	0.0072	-0.0156
	5	0.0578	0.0210	0.0424	0.0623	-0.0305	-0.0359	-0.0349	0.0045	0.0069	-0.0052
	10	0.0602	0.0181	0.0372	0.0787	-0.0275	-0.0362	-0.0470	0.0040	0.0072	-0.0065
	20	0.0613	0.0222	0.0345	0.0755	-0.0262	-0.0365	-0.0468	0.0039	0.0069	-0.0052
	40	0.0630	0.0240	0.0300	0.0750	-0.0239	-0.0368	-0.0464	0.0035	0.0068	-0.0051
	80	0.0610	0.0241	0.0375	0.0723	-0.0287	-0.0376	-0.0444	0.0043	0.0070	-0.0047
G_3	**2**	0.0355	-0.0036	0.0097	0.0544	-0.0114	-0.0164	-0.0208	0.0018	0.0042	-0.0087
	5	0.0336	0.0117	0.0142	0.0395	-0.0142	-0.0205	-0.0259	0.0023	0.0040	-0.0029
	10	0.0349	0.0105	0.0116	0.0483	-0.0127	-0.0209	-0.0336	0.0020	0.0041	-0.0032
	20	0.0355	0.0131	0.0099	0.0461	-0.0119	-0.0212	-0.0337	0.0019	0.0039	-0.0022
	40	0.0365	0.0141	0.0073	0.0460	-0.0106	-0.0214	-0.0338	0.0017	0.0039	-0.0021
	80	0.0354	0.0140	0.0116	0.0445	-0.0134	-0.0218	-0.0326	0.0022	0.0040	-0.0019
G_4	**2**	0.0234	-0.0020	0.0022	0.0371	-0.0058	-0.0108	-0.0166	0.0010	0.0027	-0.0054
	5	0.0222	0.0077	0.0049	0.0271	-0.0076	-0.0134	-0.0201	0.0013	0.0026	-0.0015
	10	0.0230	0.0070	0.0032	0.0328	-0.0066	-0.0137	-0.0255	0.0011	0.0026	-0.0015
	20	0.0234	0.0087	0.0021	0.0313	-0.0061	-0.0139	-0.0258	0.0011	0.0025	-0.0008
	40	0.0240	0.0093	0.0005	0.0312	-0.0052	-0.0140	-0.0260	0.0010	0.0025	-0.0007
	80	0.0233	0.0092	0.0032	0.0304	-0.0070	-0.0142	-0.0253	0.0012	0.0025	-0.0006

Results: A small crack in a thin pipe

Axial surface crack with $a/t = 0.01$, $R_i/t = 80$

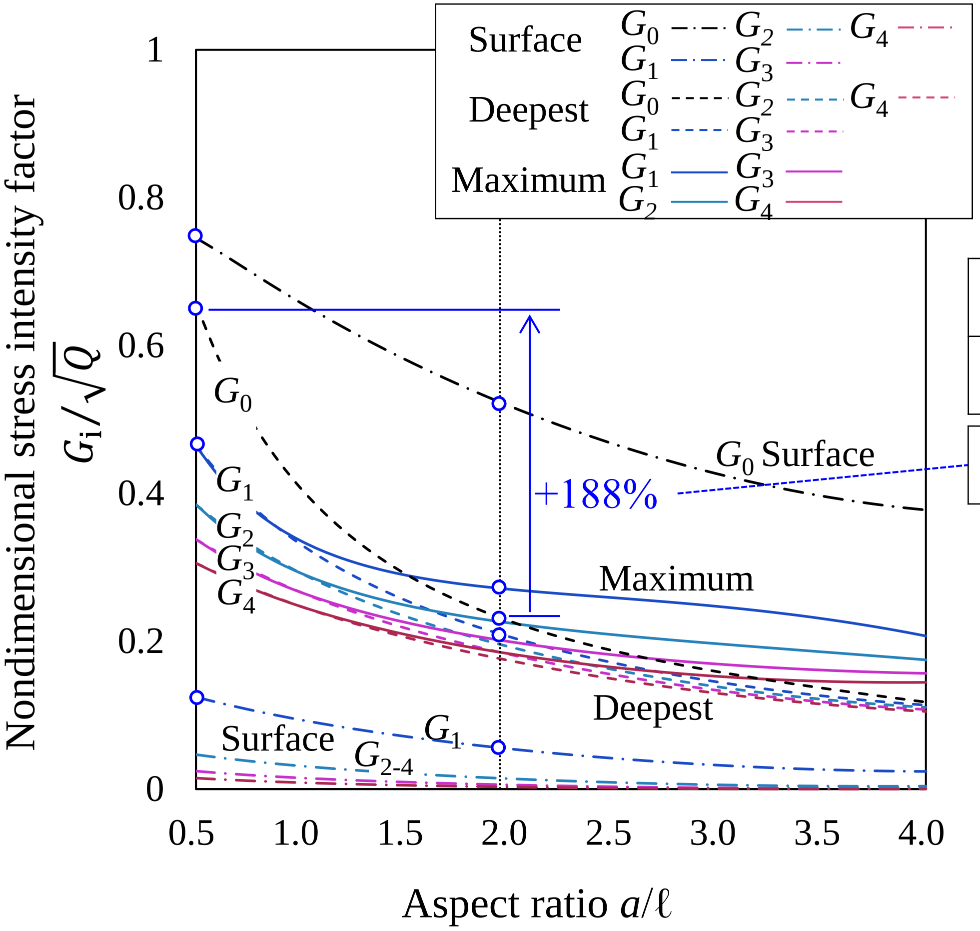


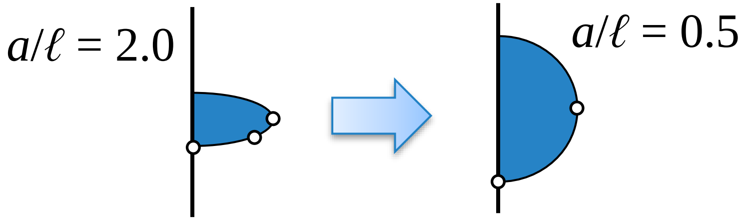


	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.519	0.746	+43.8%

Results: A small crack in a thin pipe

Axial surface crack with $a/t = 0.01$, $R_i/t = 80$

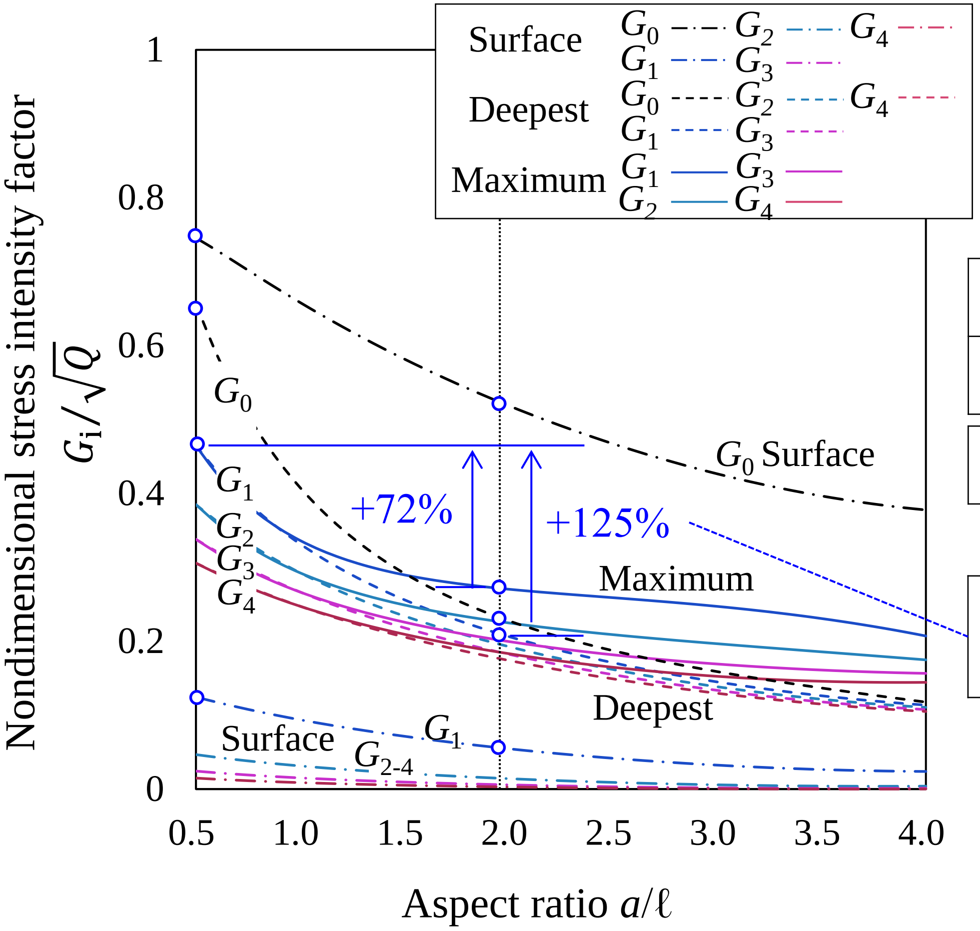


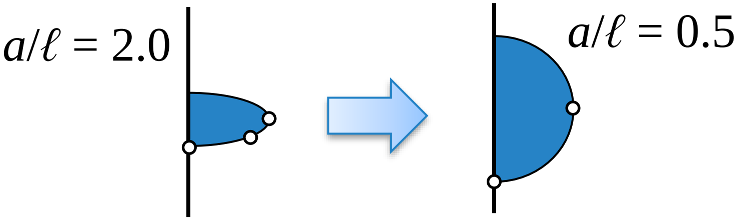


	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.519	0.746	+43.8%
G_0	Deepest	0.228	0.656	+188%

Results: A small crack in a thin pipe

Axial surface crack with $a/t = 0.01$, $R_i/t = 80$

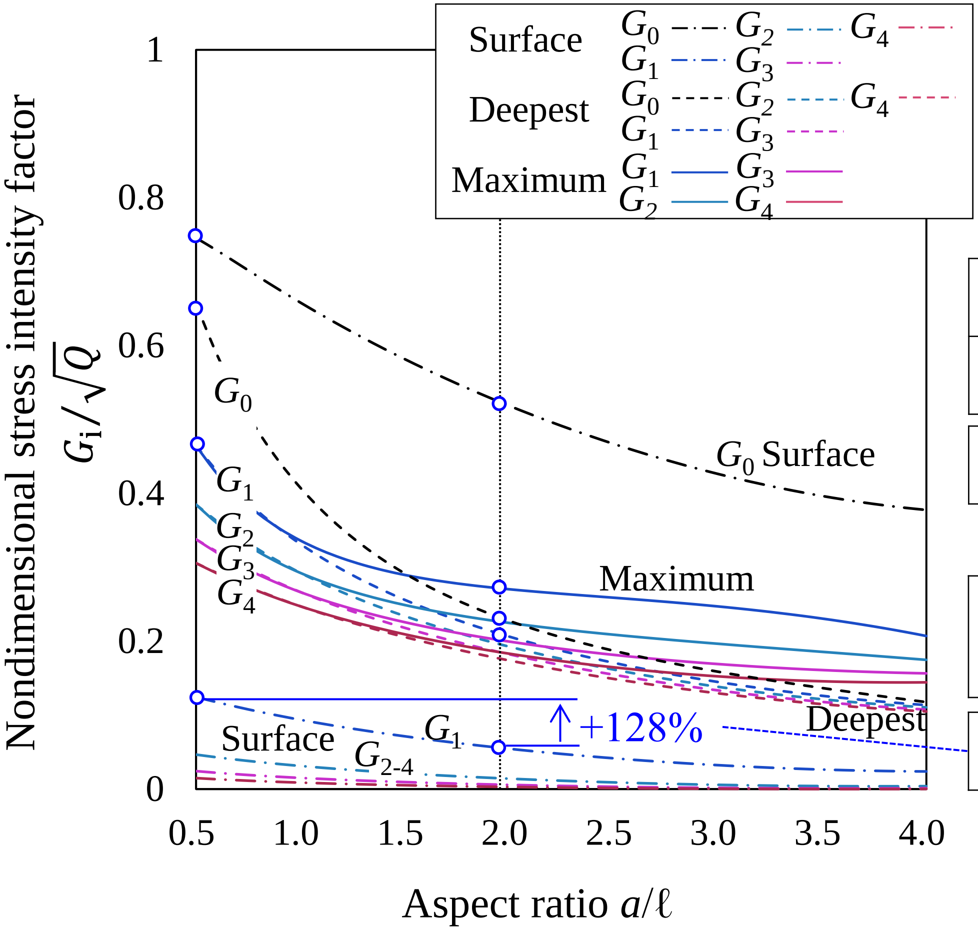


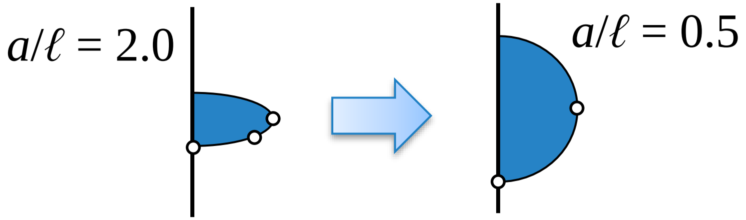


	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.519	0.746	+43.8%
G_0	Deepest	0.228	0.656	+188%
G_1	Deepest (Maximum)	0.206 (0.270)	0.464 (0.464)	+125% (+71.7%)

Results: A small crack in a thin pipe

Axial surface crack with $a/t = 0.01$, $R_i/t = 80$



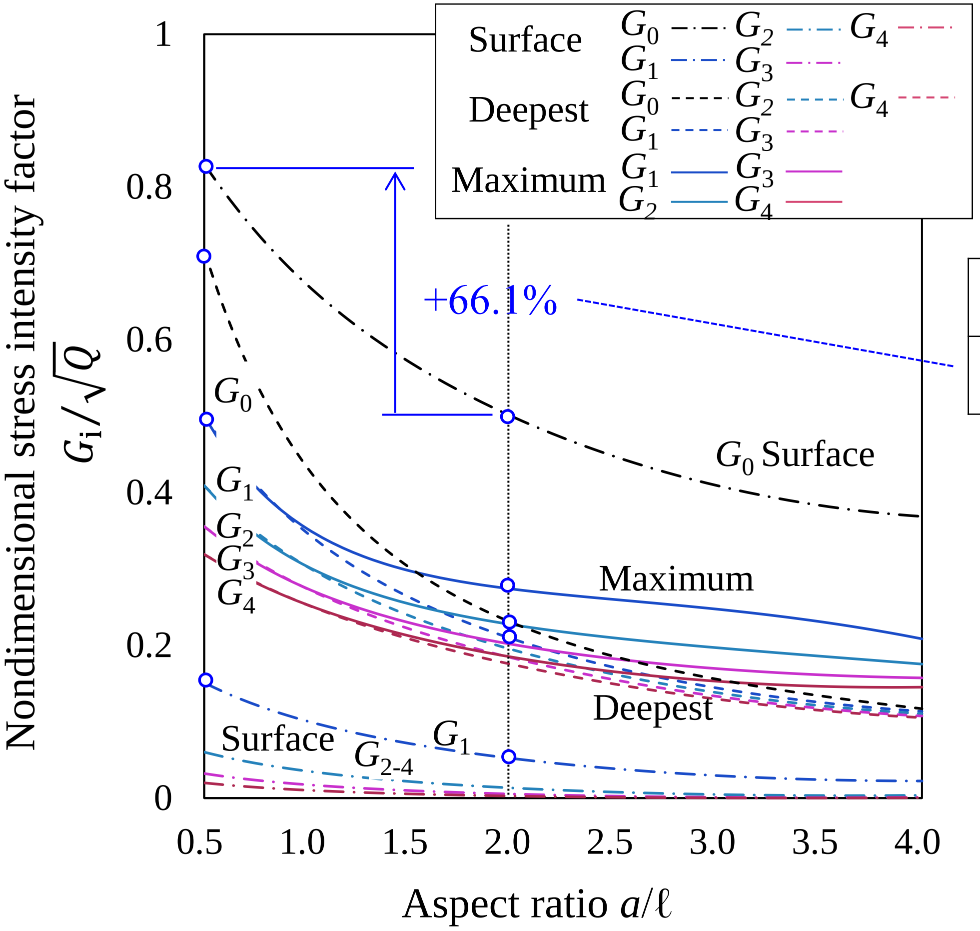


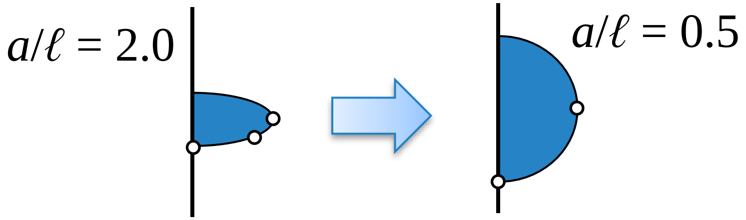
	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.519	0.746	+43.8%
G_0	Deepest	0.228	0.656	+188%
G_1	Deepest (Maximum)	0.206 (0.270)	0.464 (0.464)	+125% (+71.7%)
G_1	Surface	0.055	0.124	+128%

- Enlarging crack size always increases in G_i values.
- **The rate of increase highly depends on** the load condition, the evaluation point, and the original crack aspect ratio.

Results: A large crack in a thick pipe

Axial surface crack with *a/t* = 0.01, R_i/t = 80

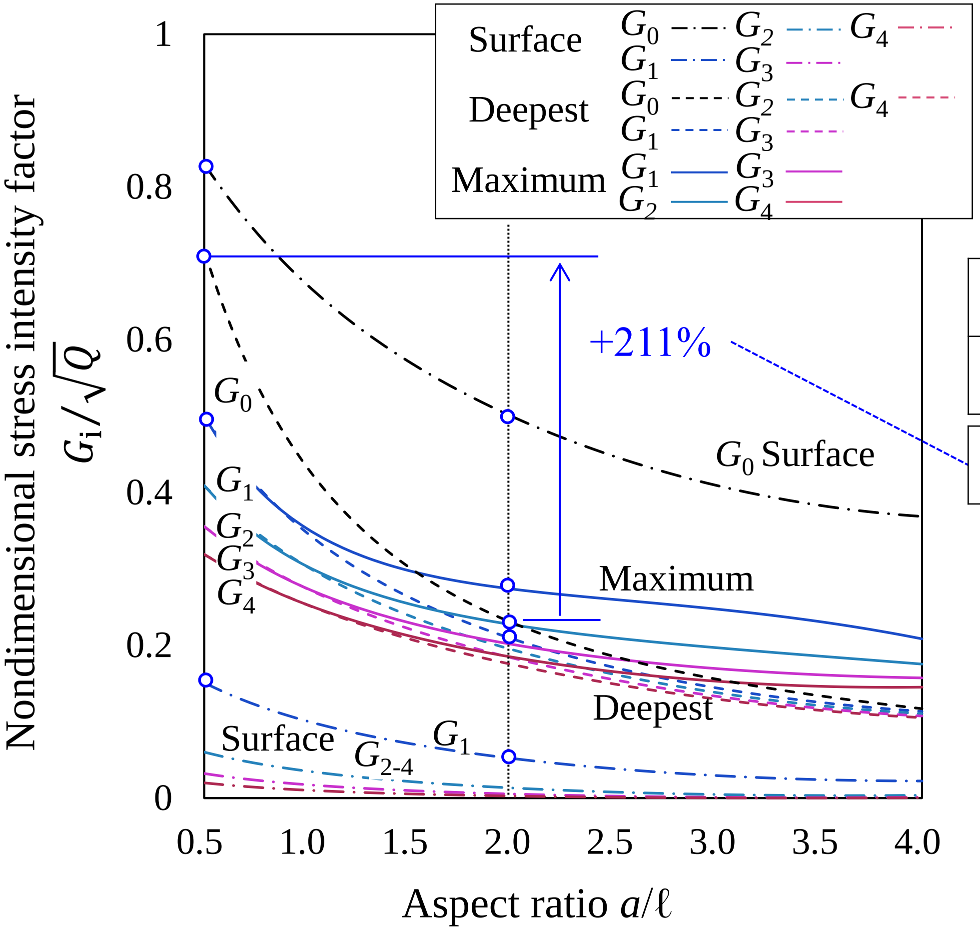


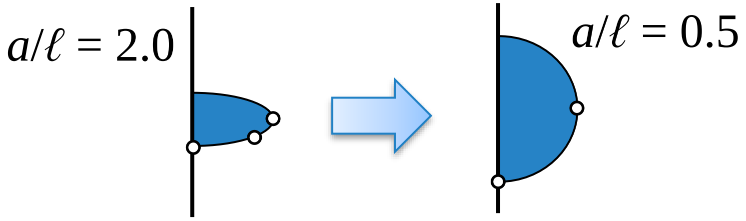


	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.500	0.830	+66.1%

Results: A large crack in a thick pipe

Axial surface crack with $a/t = 0.01$, $R_i/t = 80$

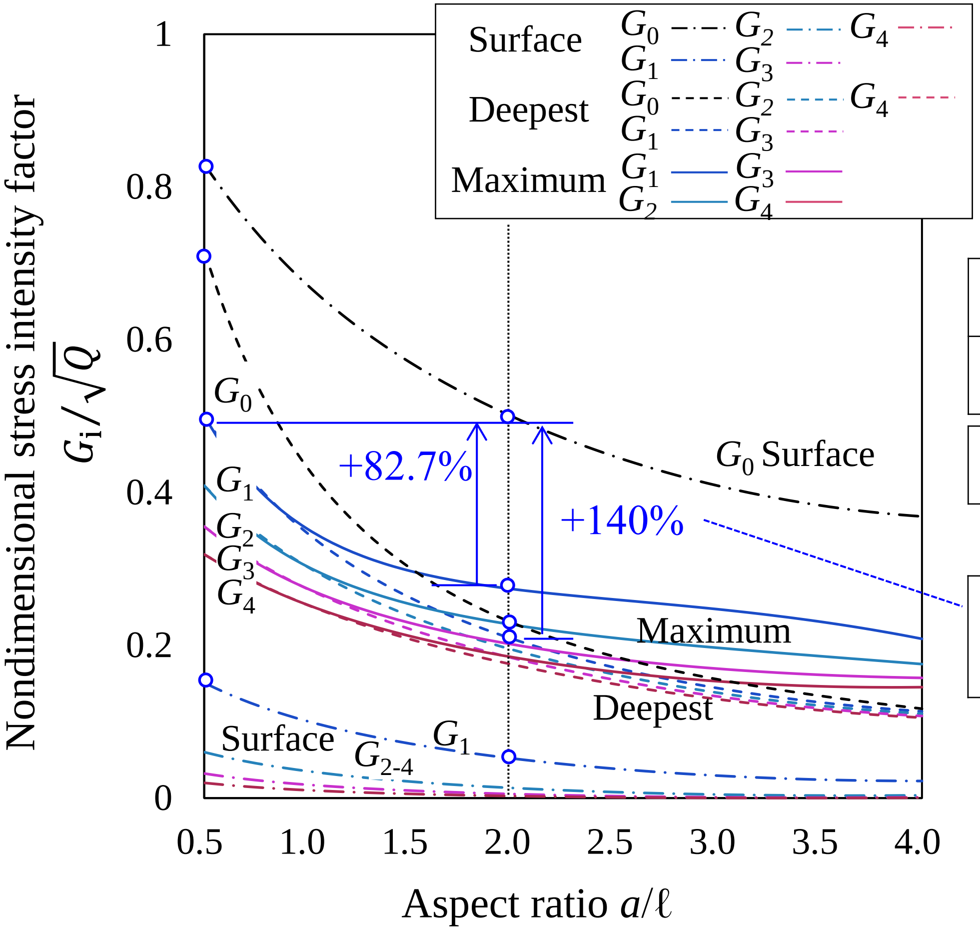


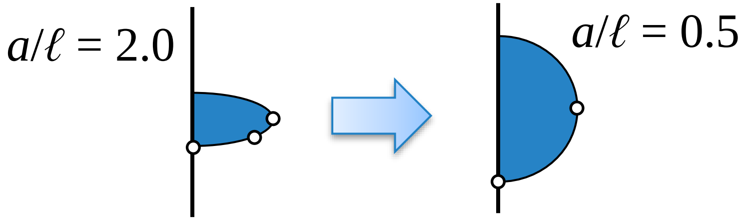


	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.500	0.830	+66.1%
G_0	Deepest	0.230	0.716	+211%

Results: A large crack in a thick pipe

Axial surface crack with $a/t = 0.01$, $R_i/t = 80$

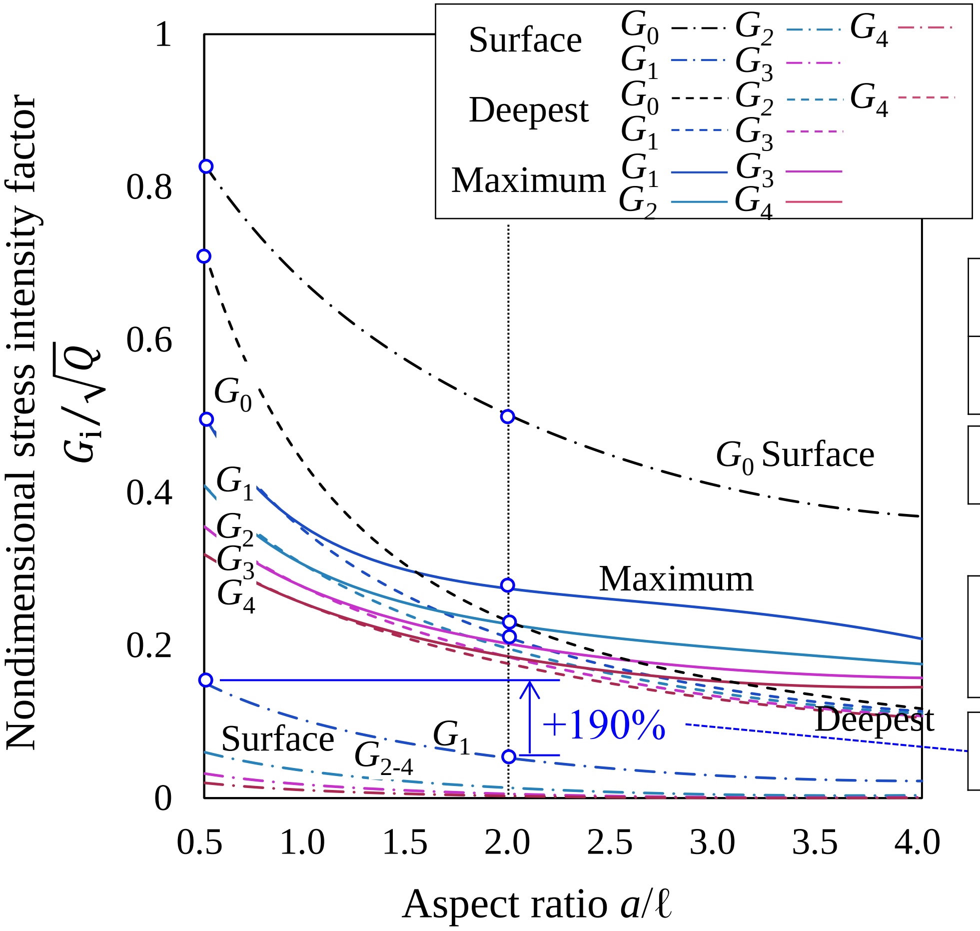


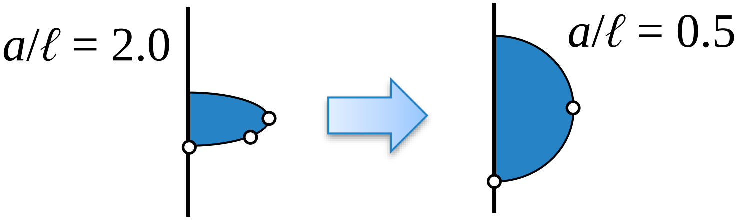


	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.500	0.830	+66.1%
G_0	Deepest	0.230	0.716	+211%
G_1	Deepest (Maximum)	0.209 (0.274)	0.500 (0.500)	+140% (+82.7%)

Results: A large crack in a thick pipe

Axial surface crack with $a/t = 0.01$, $R_i/t = 80$





	Evaluation points	$a/\ell = 2.0$	$a/\ell = 0.5$	% increase
G_0	Surface	0.500	0.830	+66.1%
G_0	Deepest	0.230	0.716	+211%
G_1	Deepest (Maximum)	0.209 (0.274)	0.500 (0.500)	+140% (+82.7%)
G_1	Surface	0.052	0.151	+190%

- Enlarging crack size always results in an increase in G_i values.
- **The rate of increase highly depends on** the load condition, the evaluation point, and the original crack aspect ratio.

Summary

We developed the closed-form G_0-G_4 solutions for surface cracks with $a/\ell \geq 0.5$ in pipes.

- The closed-form equations for axial and circumferential cracks are sufficiently precise (The max. error is 8% at the surface point).
- G_i values depend mainly on aspect ratio a/ℓ rather than crack depth ratio a/t and radius/thickness ratio R_i/t.
- Enlarging cracks with $a/\ell \geq 0.5$ always increases G_i values; the rate of increase depends on the load condition, the evaluation point, and the original crack aspect ratio.

The degree of conservativeness varies a lot.
This might introduce uncertainties in probabilistic analysis.