

# Uncertainty Propagation by Nested and Non-Nested Sampling in Probabilistic Fitness-For-Service Evaluations of Pressure Tubes in CANDU Reactors

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**Fourth International Symposium on  
Probabilistic Methodologies for Nuclear Applications**

**Leicester, UK  
November 1 - 3, 2022**

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- Pilot Study on Application of Nested Sampling to Uncertainty Propagation in Probabilistic Fitness-For-Service Evaluations
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- Conclusions

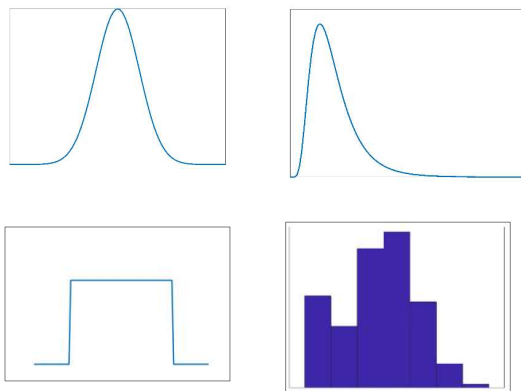
# Introduction

- CSA N285.8 is a Canadian nuclear standard that specifies the technical requirements for the owner/operator to evaluate cold-worked Zr-2.5%Nb alloy pressure tubes in CANDU reactors for continued operation.
- An informative Annex to CSA N285.8 was developed to provide guidelines for performing uncertainty analysis in probabilistic evaluations relevant to the scope of the Standard.
- The Annex does not provide guidance regarding use of either nested or non-nested sampling for the propagation of uncertainties.
  - Non-nested sampling has traditionally been used.
- A pilot study on application of nested sampling to uncertainty propagation in pressure tube fitness-for-service evaluations has been performed.

# CSA Standard N285.8 Annex on Uncertainty Analysis

- CSA N285.8 methodology for performing uncertainty analysis
  - Identification of influential variables
  - Characterization of uncertainties
  - Characterization of statistical correlations
  - Propagation of uncertainties
- No provisions with respect to using nested or non-nested sampling

## Uncertainties in Influential Variables

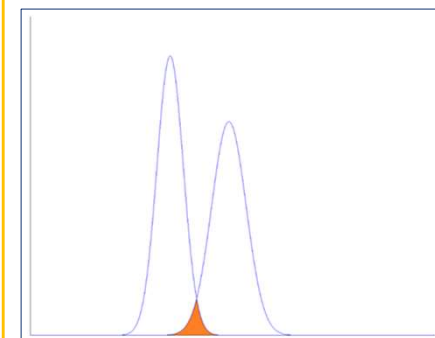


Correlated Sampling

Evaluation Procedure

Post Processing

## Evaluation Result



# CSA Standard N285.8 Annex on Uncertainty Analysis

## Uncertainty components in model response

| Variable type | Best estimate obtained using                    | Uncertainty component |   |
|---------------|---|-----------------------|---|
| Type A        | Parametric model<br><i>“mechanistic models”</i> | $U^{(p)}$             | Parametric uncertainty                                  |
|               |   | $U^{(n)}$             | Uncertainty in numerical representation                 |
| Type B        | Statistical model<br><i>“empirical models”</i>  | $U^{(r)}$             | Residual uncertainty                                    |
|               |   | $U^{(d)}$             | Uncertainty due to limitations in model-basis data sets |

No guidance is provided for characterizing model form uncertainty. Research and development work is still on-going to establish approach(es) to characterizing this uncertainty component.

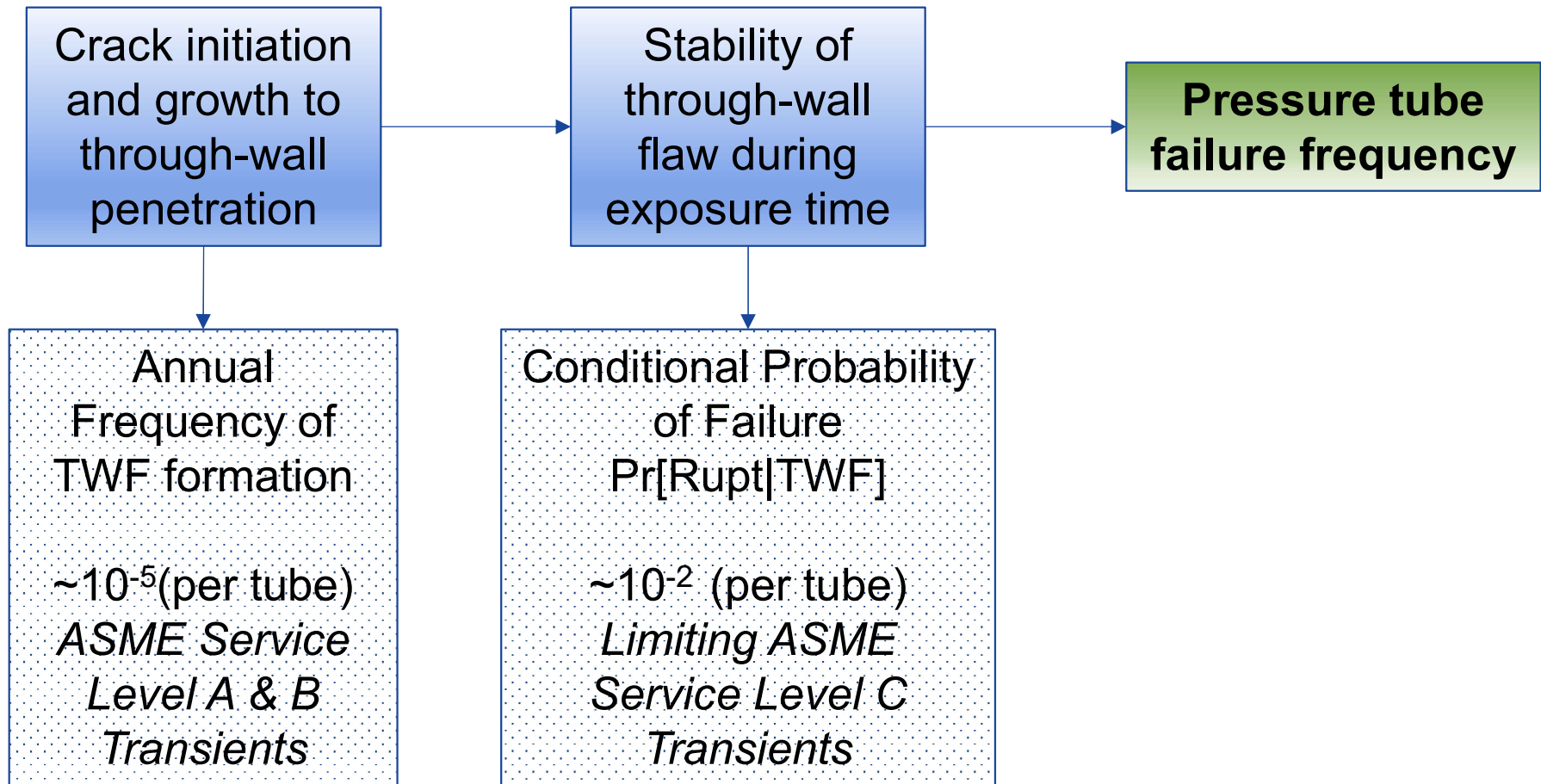
# Pilot Study on Nested Sampling Uncertainty Propagation

## Scope & Objectives

- Pilot study on application of nested sampling to uncertainty propagation
  - Portion of probabilistic evaluation of pressure tube fracture protection where a through-wall flaw is postulated to exist
  - The result of this portion of the evaluation is the conditional probability of failure given the existence of a through-wall flaw
- The objectives were:
  - Investigate impact of nested sampling on computer code performance
  - Investigate impact of nested sampling on evaluation outcome

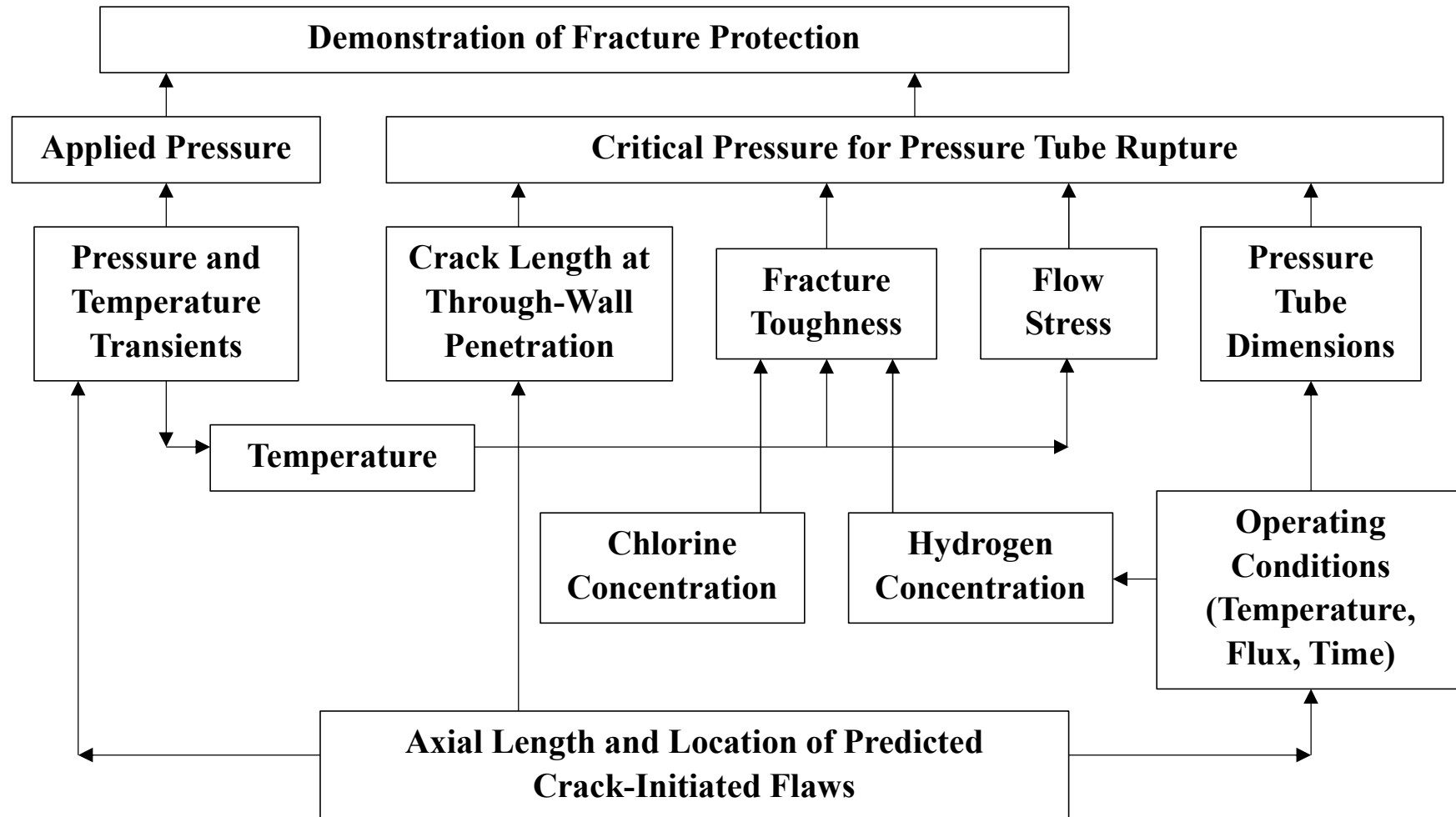
# Pilot Study on Nested Sampling Uncertainty Propagation

## Probabilistic Fracture Protection Evaluation



# Pilot Study on Nested Sampling Uncertainty Propagation

## Probabilistic Fracture Protection Evaluation





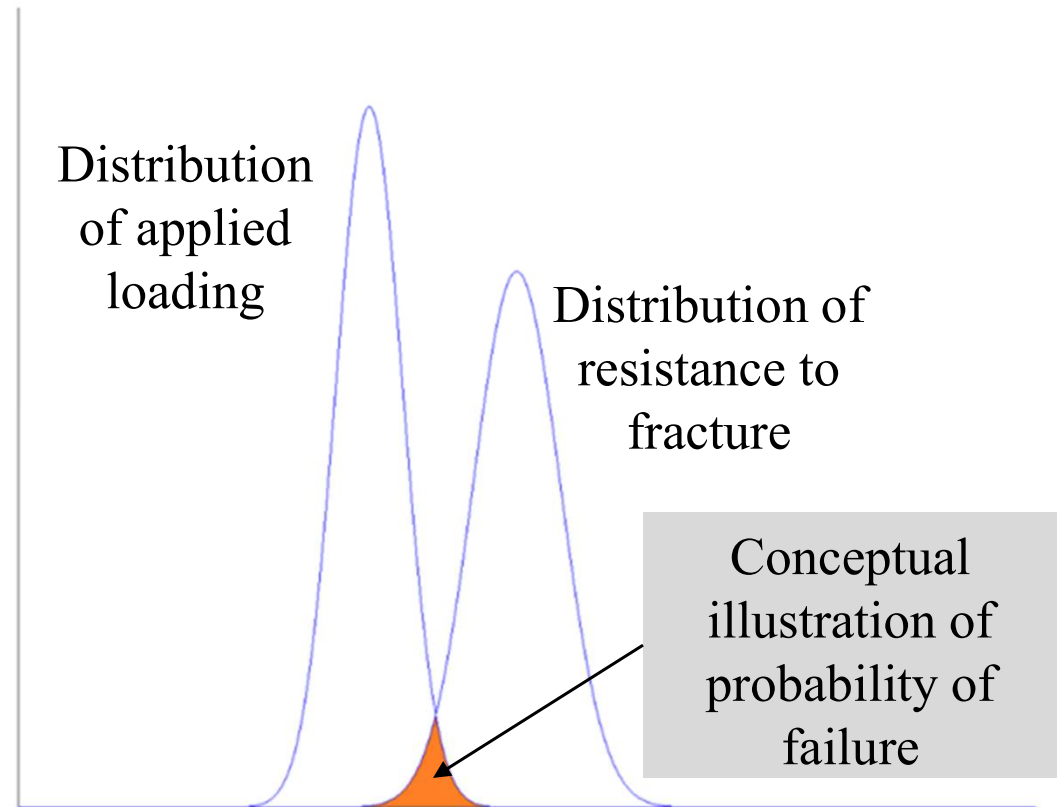
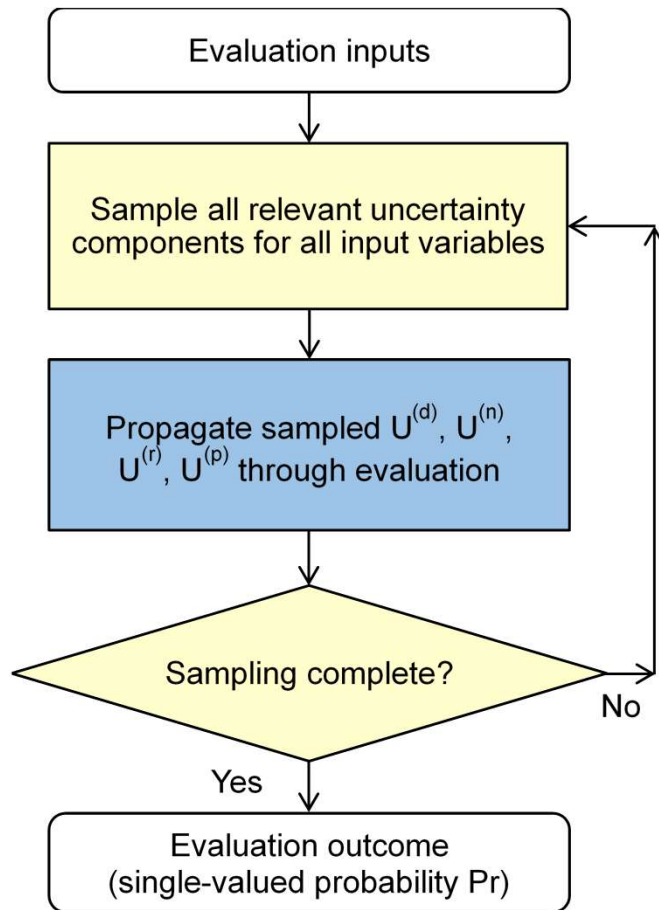
# Pilot Study on Nested Sampling Uncertainty Propagation

## Methodology

- Uncertainties in input variables were first propagated using non-nested sampling (traditional approach)
  - All uncertainty components were propagated together in a single loop
  - Conditional probability of pressure tube rupture obtained as a single-valued quantity
  - Approximately 55,000 simulations were performed using random sampling

# Pilot Study on Nested Sampling Uncertainty Propagation

## Methodology



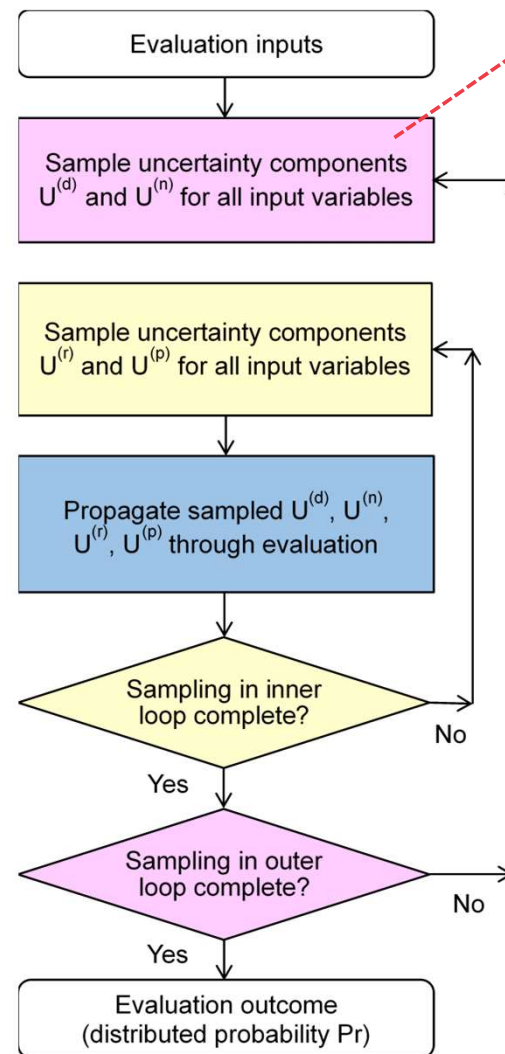
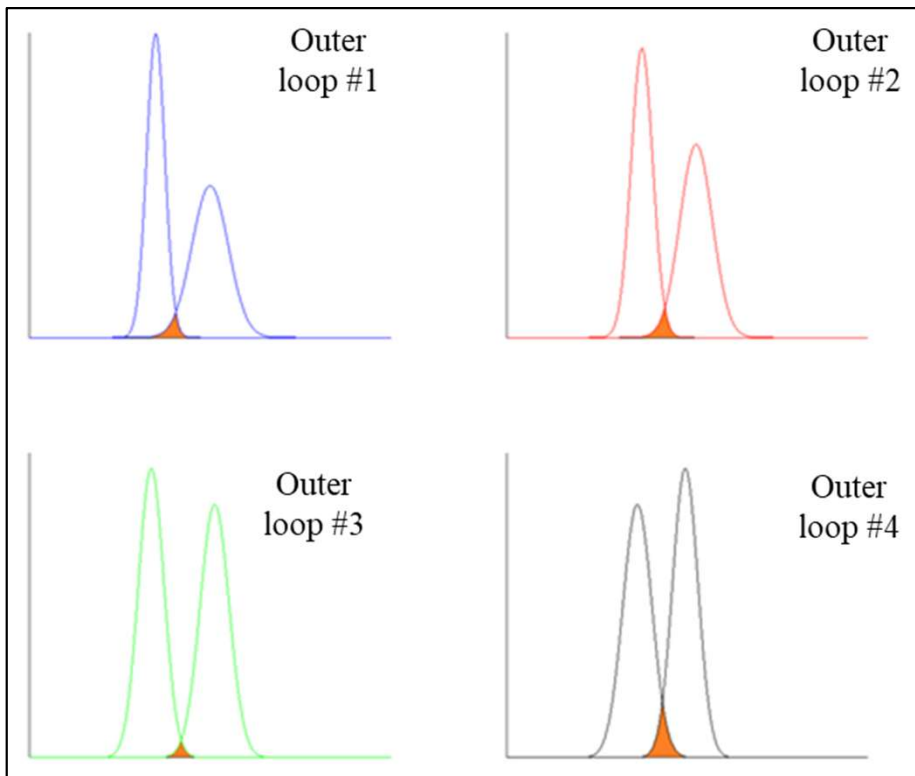
# Pilot Study on Nested Sampling Uncertainty Propagation

## Methodology

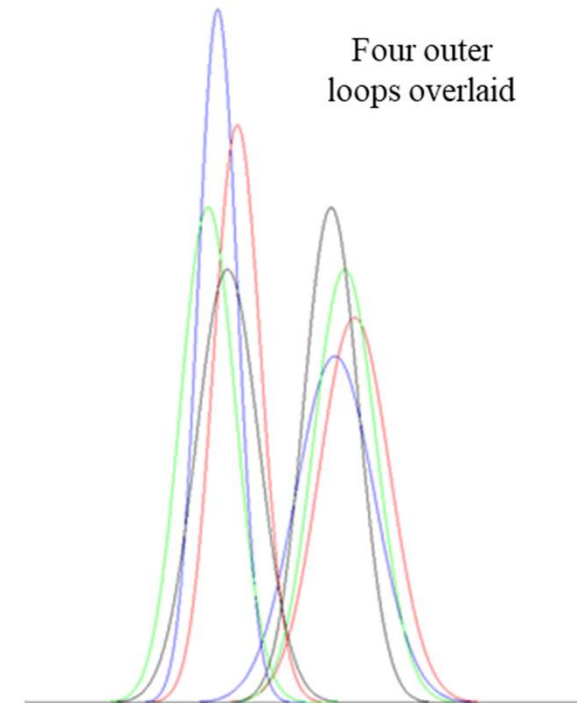
- Uncertainties in input variables were next propagated using nested sampling
  - Uncertainty components were segregated between an inner loop and an outer loop
  - Uncertainty components that are predominantly epistemic in nature were placed in the outer loop (as is commonly done in nested sampling).
  - Conditional probability of pressure tube rupture obtained as a distributed quantity
  - Approximately 55,000 simulations were performed using random sampling for the inner loop
  - 1,000 simulations were performed using random sampling for the outer loop

# Pilot Study on Nested Sampling Uncertainty Propagation

## Methodology



Uncertainty components that are generally more epistemic in nature



# Pilot Study on Nested Sampling Uncertainty Propagation

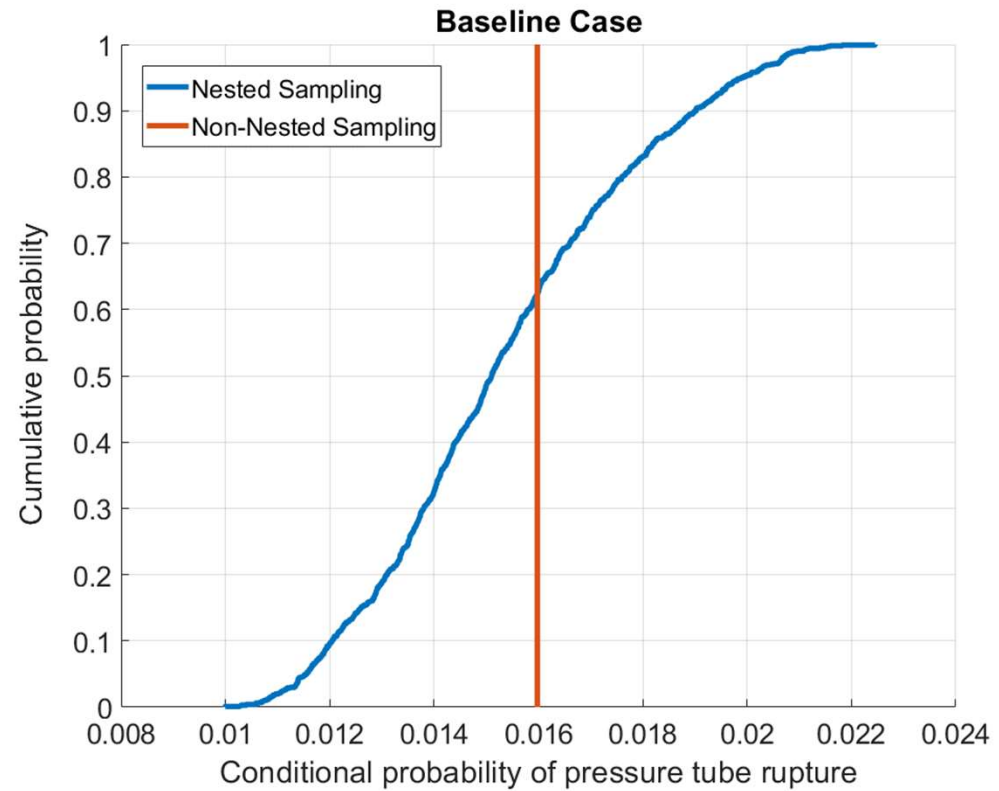
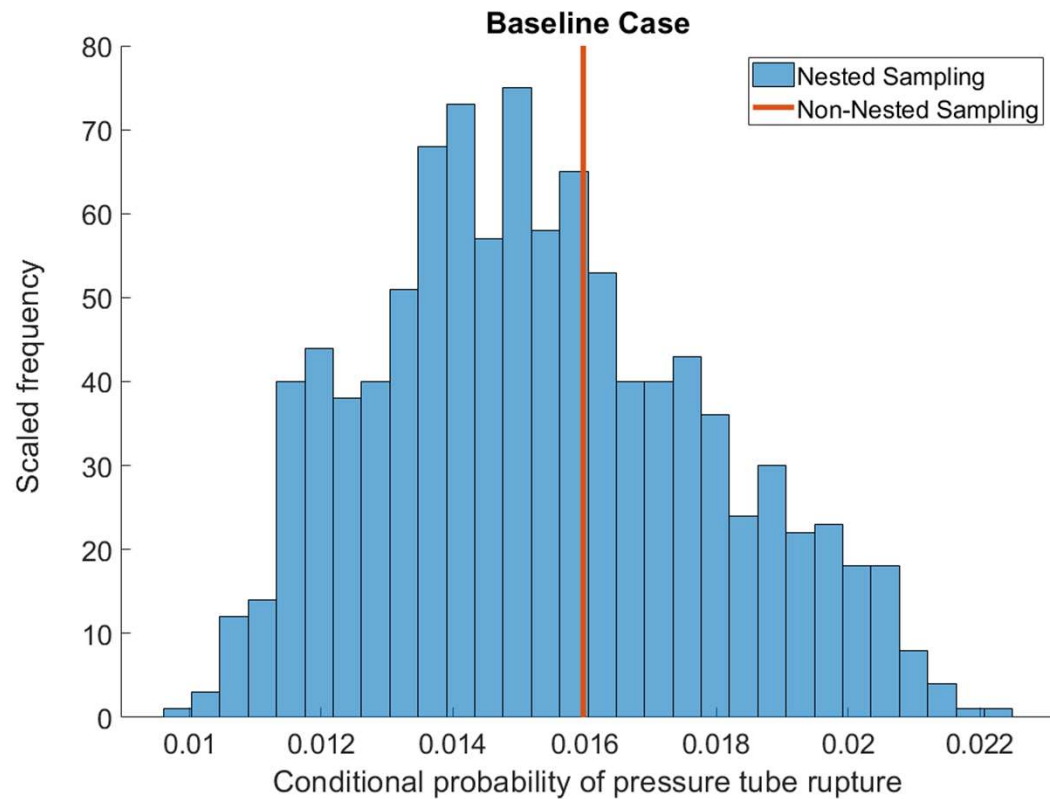
## Methodology

- Two evaluation streams were considered, baseline and extended
- In the baseline evaluation case, uncertainty components were adopted from relevant predictive models and previous work on uncertainty characterization
- In the extended evaluation cases:
  - Uncertainty components in the inner loop were the same as in baseline case
  - Uncertainty components in the outer loop were postulated such that results of the pilot study would be applicable to a range of possible uncertainty characterizations

# Pilot Study on Nested Sampling Uncertainty Propagation

## Results

### Limiting Channel, Service Level C Loadings

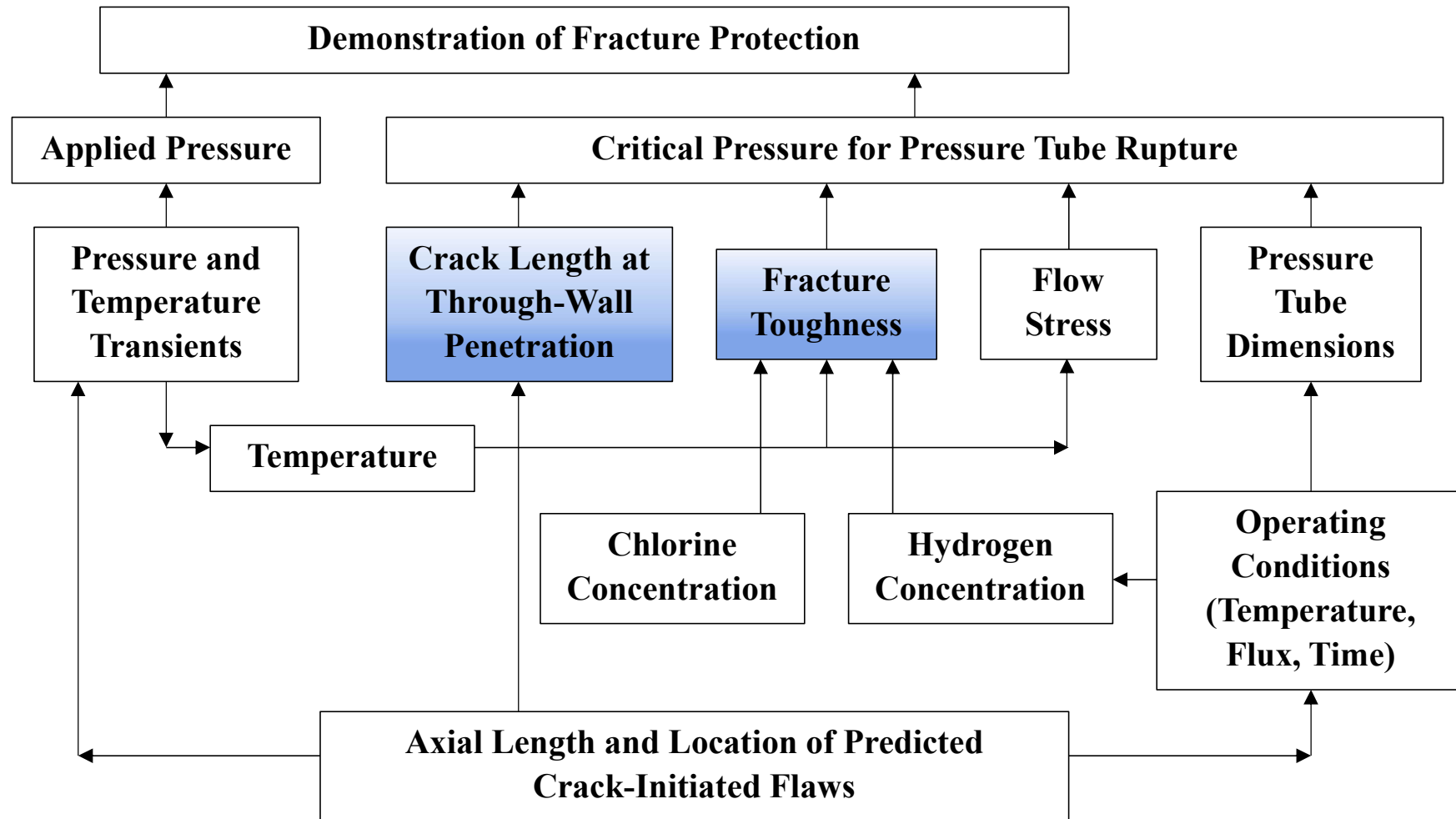


| $\Pr_{NN}$ | $\Pr_{IO} (50\%)$ | $\Pr_{IO} (95\%)$ | $\Pr_{IO} (97.5\%)$ | $Q_{NN}$ | $\frac{\Pr_{IO} (50\%)}{\Pr_{NN}}$ | $\frac{\Pr_{IO} (95\%)}{\Pr_{NN}}$ | $\frac{\Pr_{IO} (97.5\%)}{\Pr_{NN}}$ |
|------------|-------------------|-------------------|---------------------|----------|------------------------------------|------------------------------------|--------------------------------------|
| 0.01598    | 0.01510           | 0.01989           | 0.02064             | 61%      | 0.95                               | 1.24                               | 1.29                                 |

# Pilot Study on Nested Sampling Uncertainty Propagation

## Results

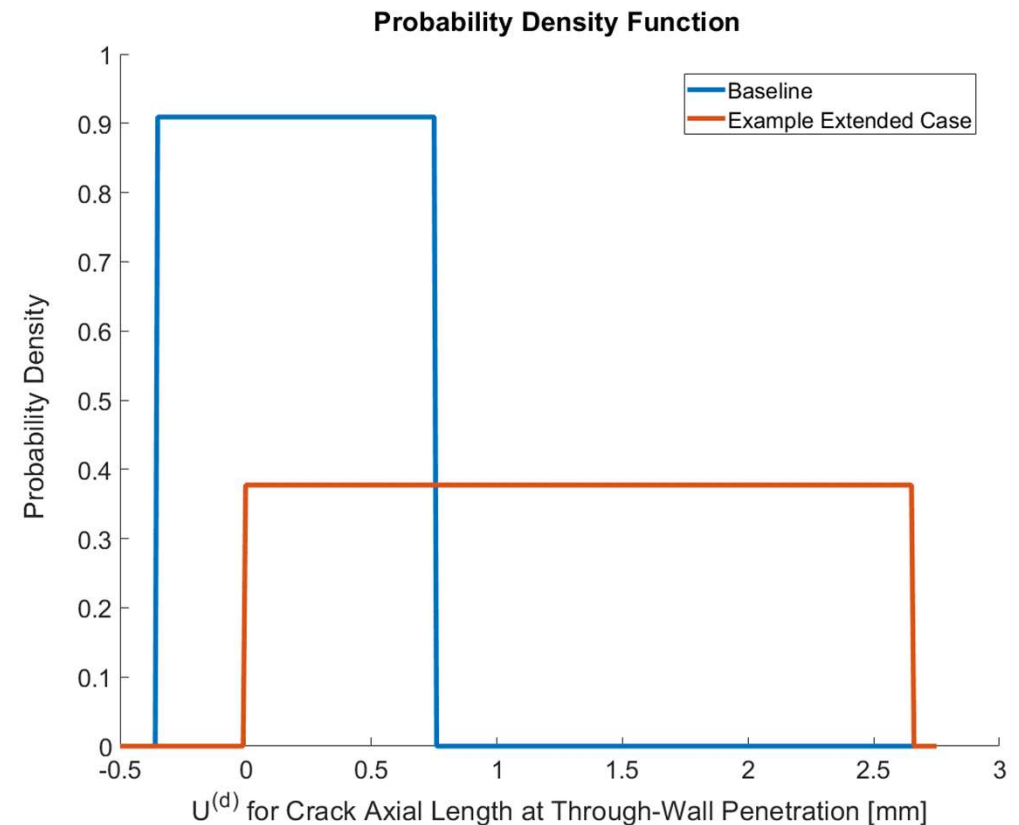
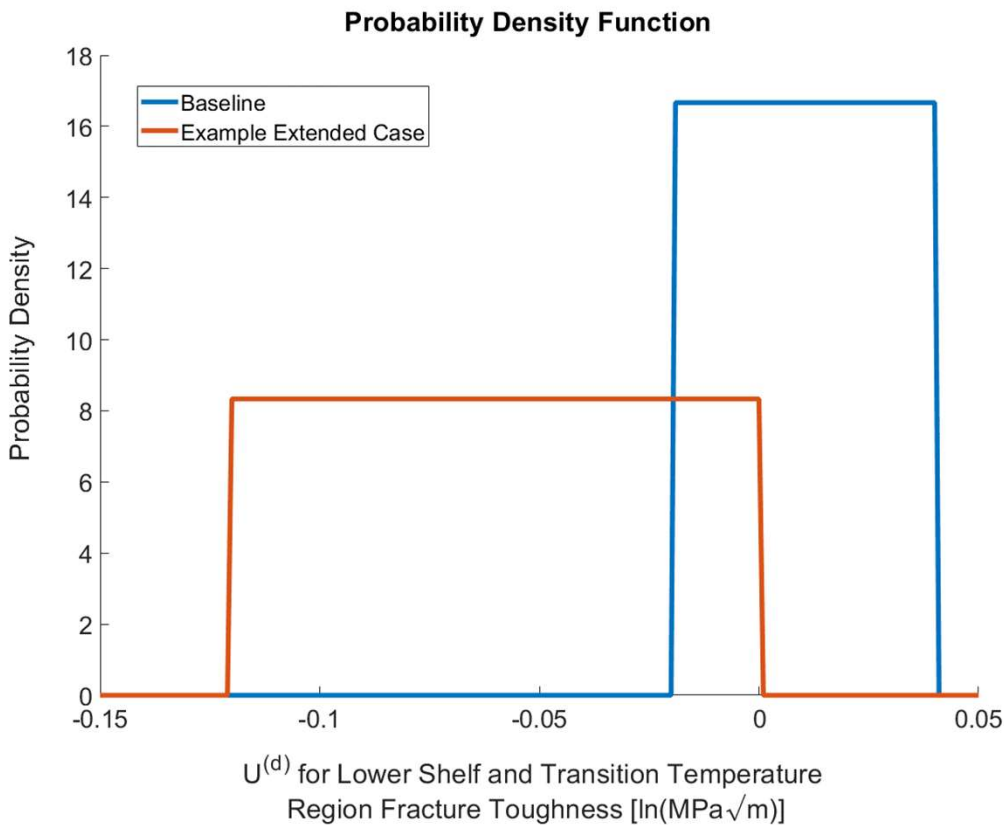
### Defining the Example Extended Case



# Pilot Study on Nested Sampling Uncertainty Propagation

## Results

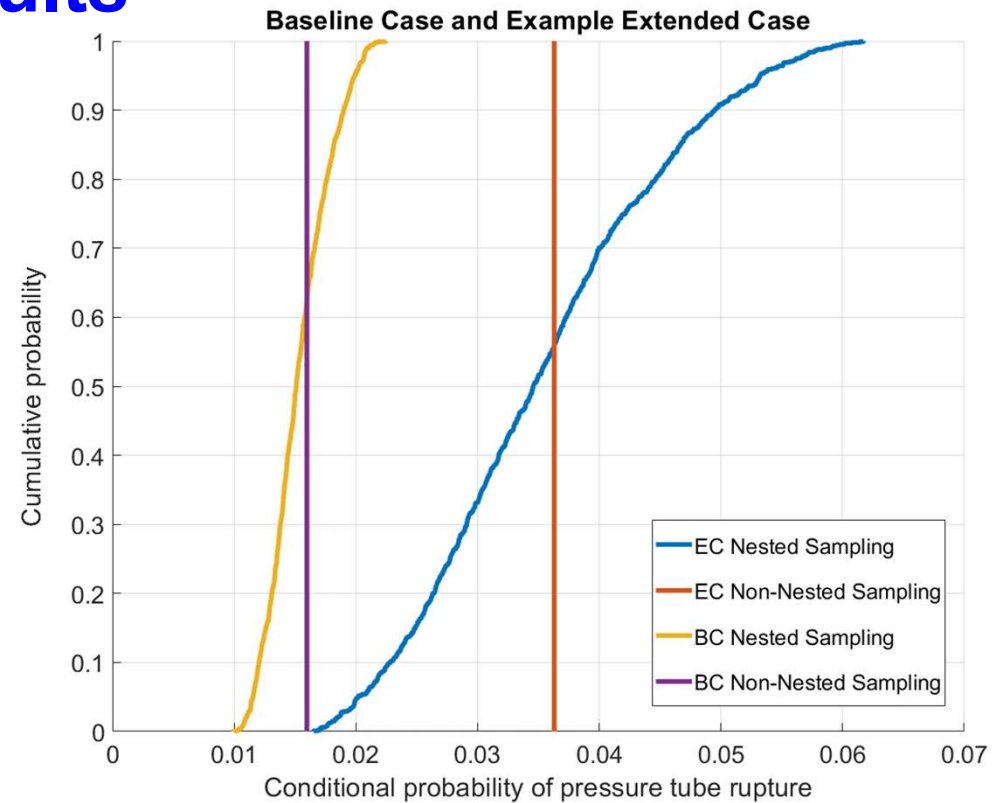
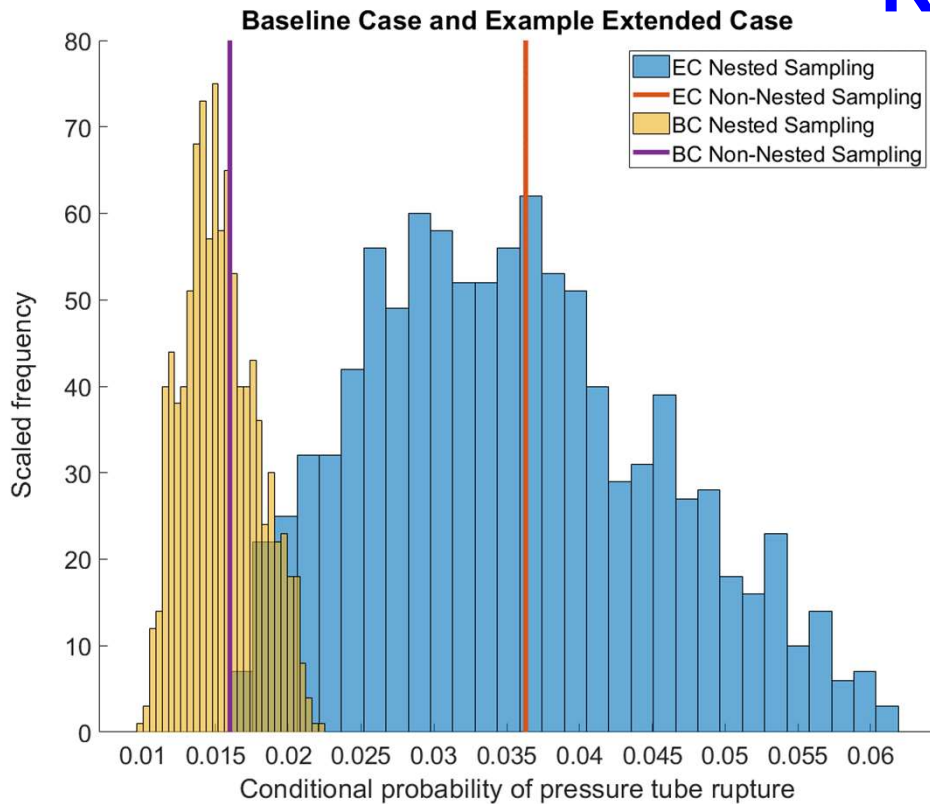
### Defining the Example Extended Case





# Pilot Study on Nested Sampling Uncertainty Propagation

## Results



| Case             | $Pr_{NN}$ | $Pr_{Io}(50\%)$ | $Pr_{Io}(95\%)$ | $Pr_{Io}(97.5\%)$ | $Q_{NN}$ | $\frac{Pr_{Io}(50\%)}{Pr_{NN}}$ | $\frac{Pr_{Io}(95\%)}{Pr_{NN}}$ | $\frac{Pr_{Io}(97.5\%)}{Pr_{NN}}$ |
|------------------|-----------|-----------------|-----------------|-------------------|----------|---------------------------------|---------------------------------|-----------------------------------|
| Baseline         | 0.01598   | 0.01510         | 0.01989         | 0.02064           | 61%      | 0.95                            | 1.24                            | 1.29                              |
| Example Extended | 0.03625   | 0.03456         | 0.05320         | 0.05617           | 56%      | 0.95                            | 1.47                            | 1.55                              |

# Conclusions

- Incorporation of nested sampling into probabilistic computer code for evaluation of fracture protection, SCEFPR, allowed the evaluation outcome (conditional probability of pressure tube rupture) to be treated as a distributed variable
  - Segregation of uncertainty components between inner loop and outer loop
  - Distributions of conditional probability of pressure tube rupture are well-behaved and unimodal
- The nested sampling approach, based on the nature of uncertainties, that was applied in the pilot study organically incorporated the framework for enhanced uncertainty analysis defined in CSA Standard N285.8

# Conclusions

- Evaluation of most limiting fuel channel, baseline case:
  - Median of distributed conditional probability was found to be about 5% lower than single-valued conditional probability
  - 97.5-th percentile of distributed conditional probability was ~1.3 times higher than single-valued conditional probability
- Evaluations of most limiting fuel channel, extended cases:
  - Median of distributed conditional probability was found to be within – 10% to + 5% of single-valued conditional probability
  - Upper-tail percentiles of distributed conditional probability with respect to single-valued conditional probability varied substantially with the magnitude of outer-loop uncertainties and their locations

# Conclusions

- Incorporation of nested sampling into computer code SCEFPR resulted in a large increase in computational time required to perform the PFP evaluation
- Improved characterization of the uncertainties associated with input variables will produce evaluation results that better reflect reality
  - Conclusion is applicable to both non-nested and nested sampling
- Nested sampling could be used for identifying areas of future research and development work
  - Distribution of evaluation outcome depends on uncertainty components that are primarily epistemic in nature