

# Accounting for Uncertainty in Complex Relationships

*Marjorie Erickson & Mark Kirk*

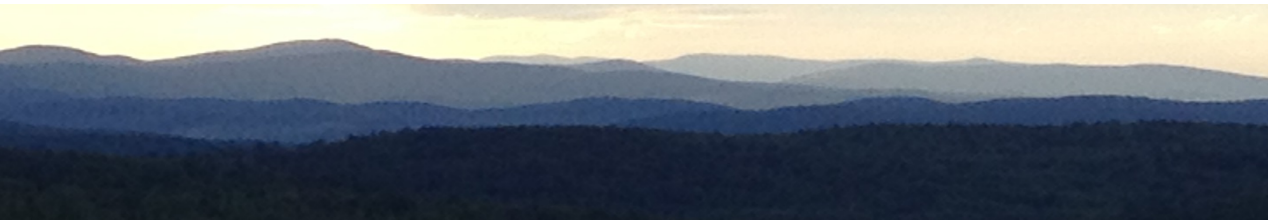
[erickson@peaiconsulting.com](mailto:erickson@peaiconsulting.com)

[kirk@peaiconsulting.com](mailto:kirk@peaiconsulting.com)

Presentation to the  
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Seminar on  
Probabilistic  
Methodologies for  
Nuclear Applications

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**Phoenix  
Engineering  
Associates  
Inc.**



# Presentation Outline

START

Background: PFM codes and uncertainty treatment

Material property models and interrelationships

Appropriate treatment of uncertainty in interrelated material property models

Uncertainty treatment in complex probabilistic code  
FAVOR

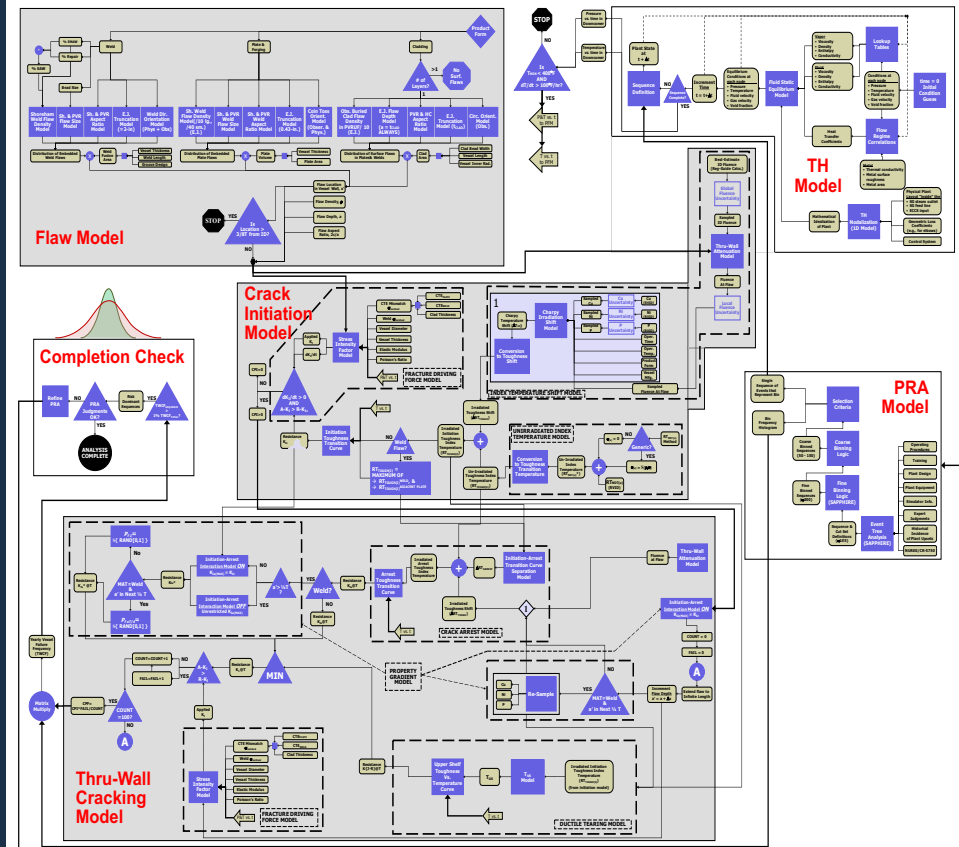
Summary & Conclusions

END



# Probabilistic Models & Codes

- Probabilistic codes are comprised of:
  - a set of models & sub models
  - linked by a framework that enables sampling
    - on distributed input variables and model parameters
    - to account for uncertainty contributions from various sources **directly and explicitly**.
- Appropriate treatment of uncertainties is key to interpreting the outcomes
  - Information flow through a code controls the cumulative impact of various sources of uncertainty on failure probabilities.
  - Sub-models must be linked together appropriately to ensure that there is no distortion of the flow of information
    - Avoid double counting of uncertainty contribution from key factors
    - Appropriately account for model bias



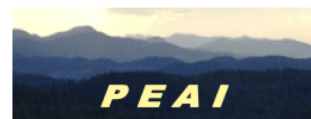
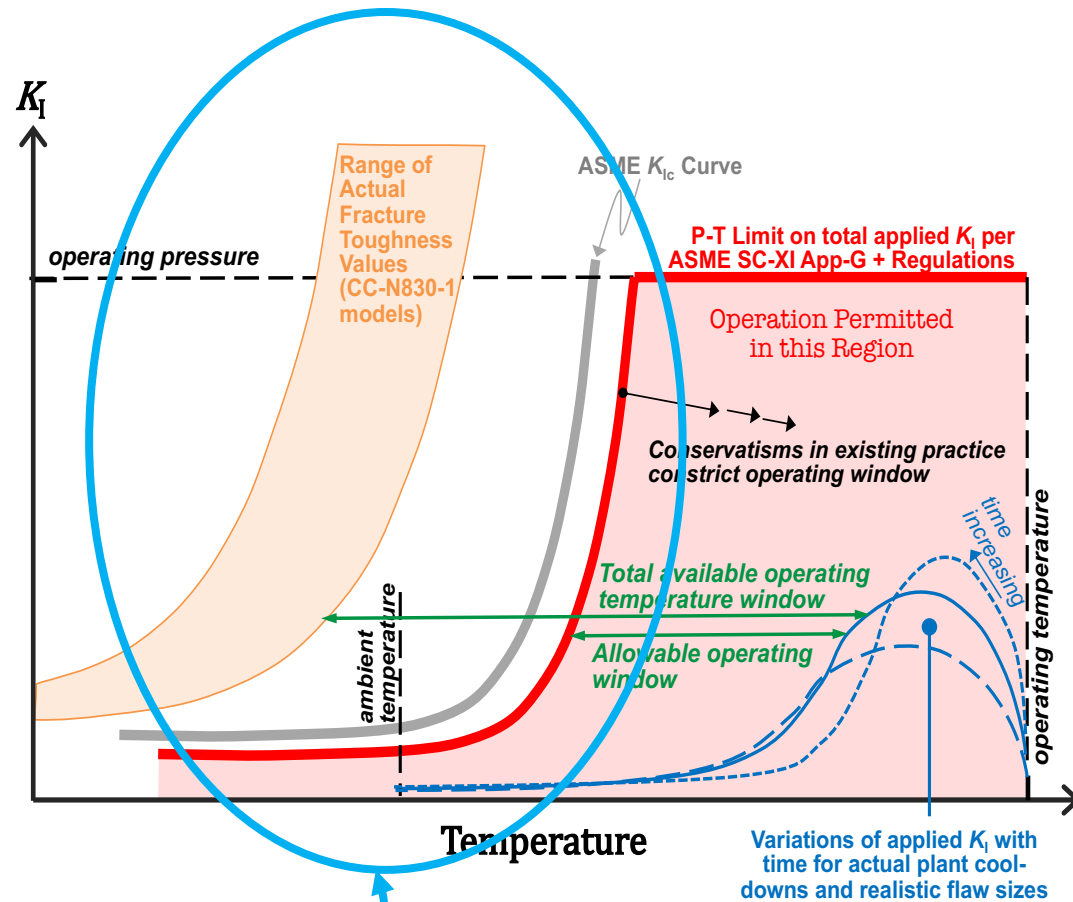
# PFM Codes

- Involve models to describe:
  - Crack driving force defined by
    - environment
    - loading
    - geometry
    - manufacturing
  - Material crack resistance defined by
    - strength
    - toughness
    - fatigue
    - SCC
  - Treatment of uncertainties that represent some phenomena of interest
  - Material crack resistance is the focus of this presentation

Driving force  $\leq$  material resistance

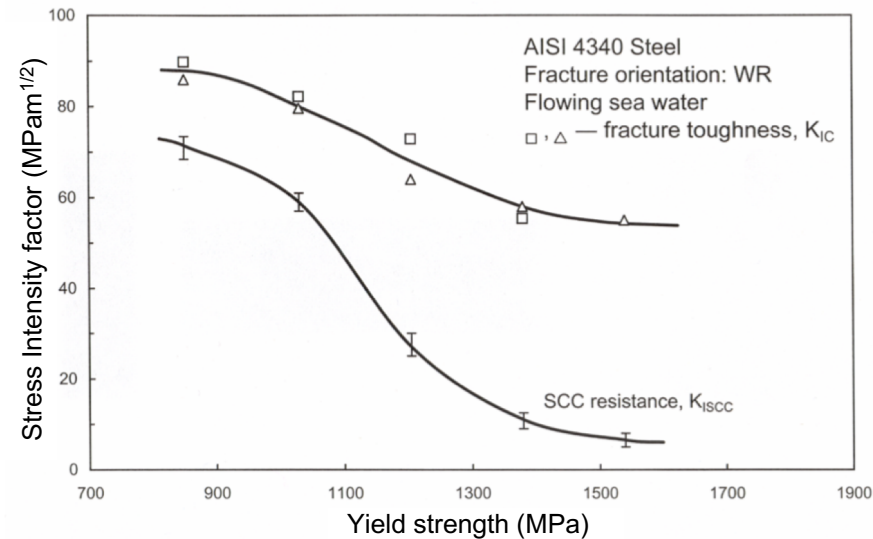
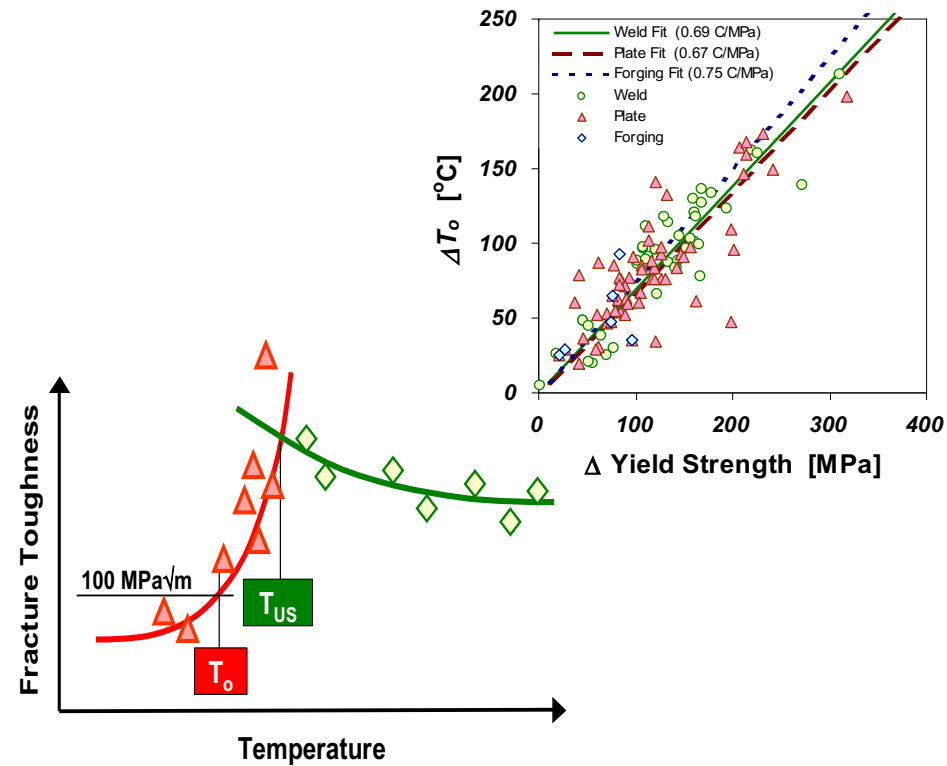
or

$$T_{\text{operating}} \geq T_{\text{cleavage toughness}}$$



# Models of Material Resistance to Failure

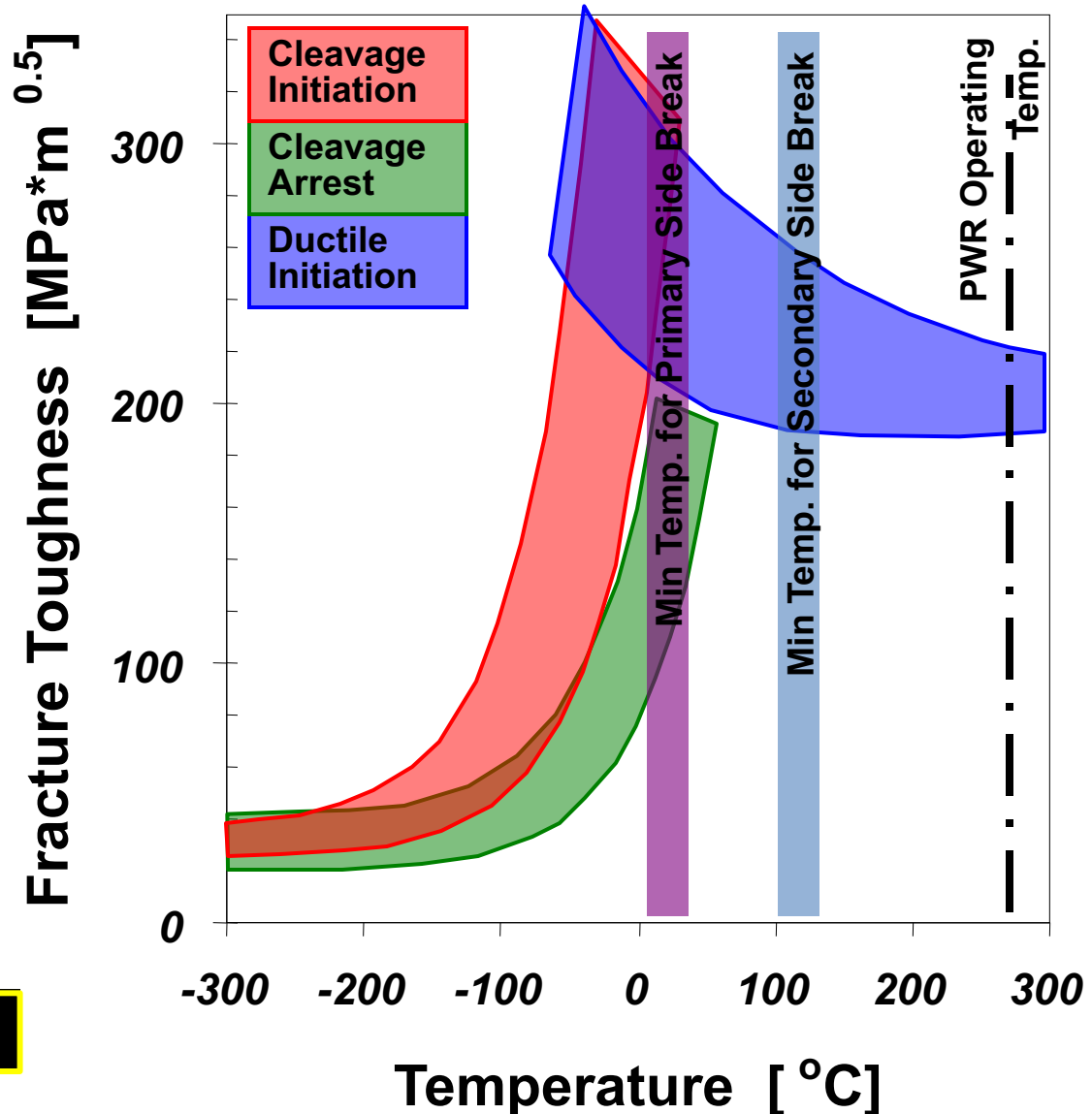
- Material resistance to through-wall cracking is characterized by various material properties:
  - Strength
  - Fracture toughness
  - Fatigue and SCC initiation and growth rates
- All of these properties are uncertain/distributed
- Many of these properties are correlated & the correlations are uncertain
- Models of properties and correlations should capture the uncertainty inherent in the data



# Focusing on Fracture Toughness: An Illustration of Linked Toughness Distributions

- The behavior of all toughness properties of interest with hardening/ embrittlement is related and can be characterized by the reference temperature ( $T_o$ )
  - Cleavage crack initiation (transition)
  - Stopping (arresting) a running cleavage crack
  - Ductile crack initiation (upper shelf)
- Example: Toughness curves for the most embrittled axial weld in a highly embrittlement plant
  - At beginning of life

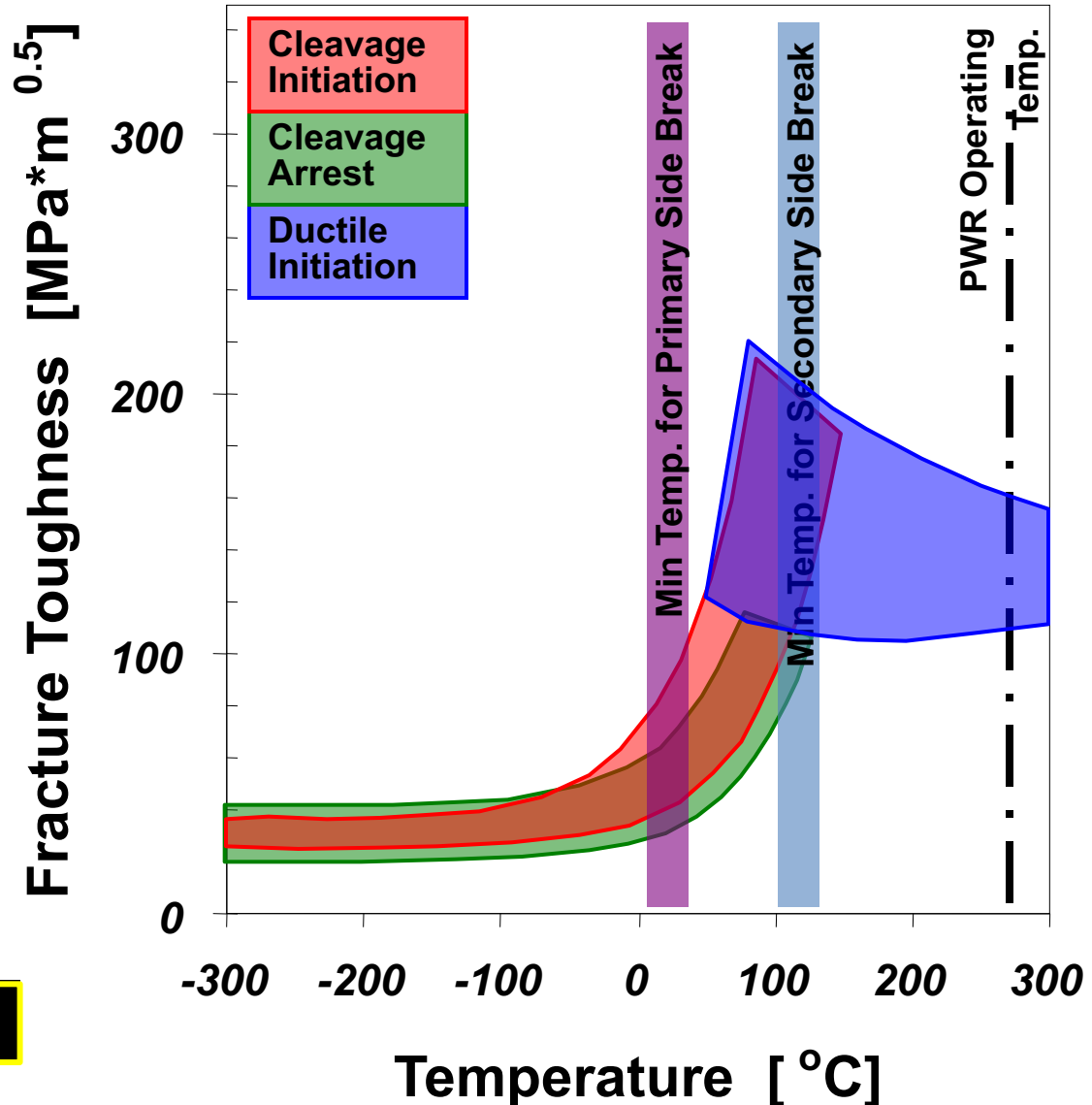
$T_o = +126^\circ\text{C}$



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- Example: Toughness curves for the most embrittled axial weld in a highly embrittlement plant
  - At beginning of life
  - At 40 years

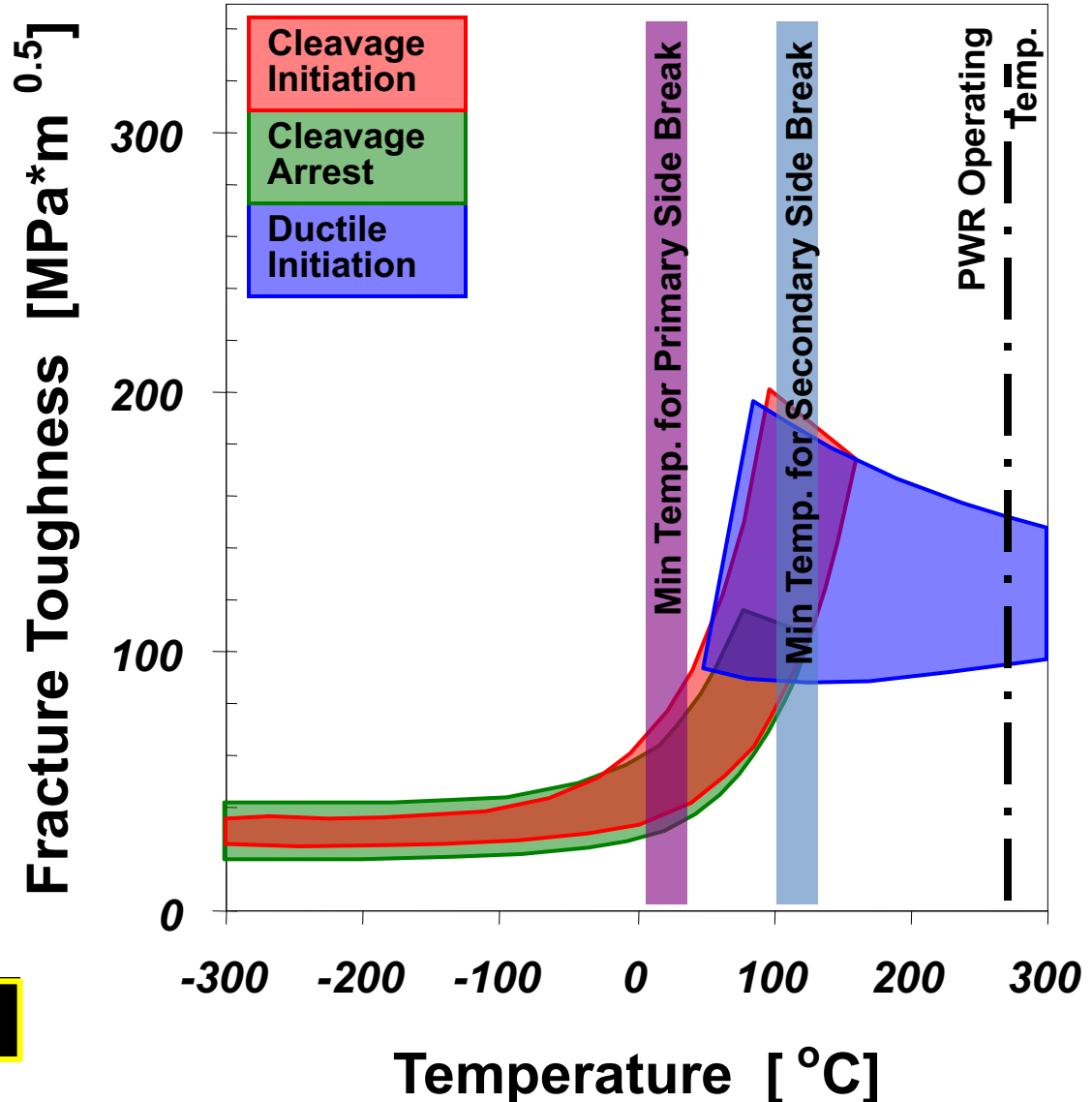
$$T_o = +126^{\circ}\text{C}$$



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- Example: Toughness curves for the most embrittled axial weld in a highly embrittlement plant
  - At beginning of life
  - At 40 years
  - At 60 years

**$T_o = +126^\circ\text{C}$**

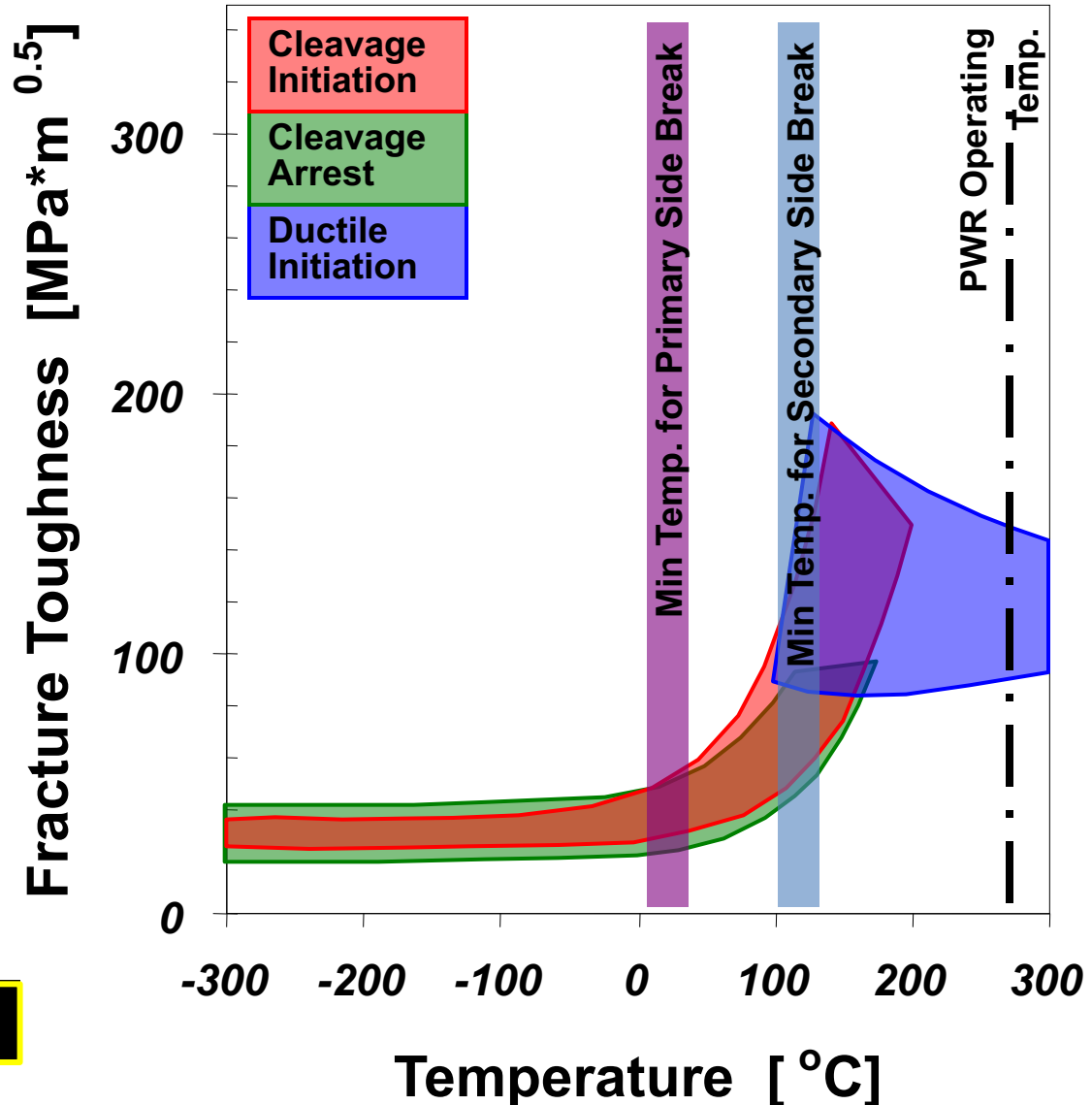




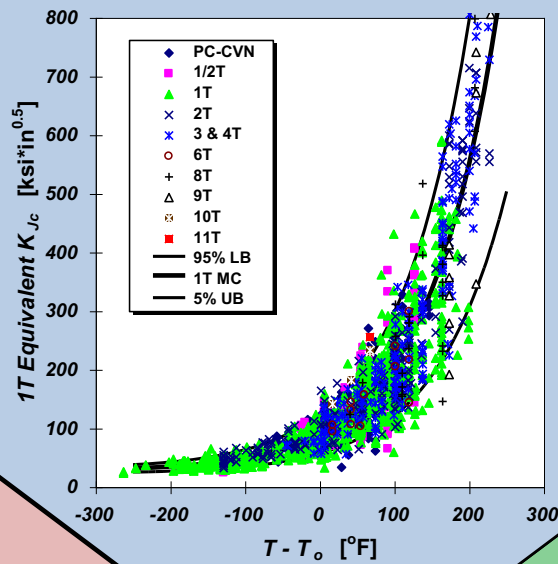
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- Example: Toughness curves for the most embrittled axial weld in a highly embrittlement plant
  - At beginning of life
  - At 40 years
  - At 60 years
  - At TWCF  $\approx 10^{-6}$  / year

$$T_o = +126^\circ\text{C}$$

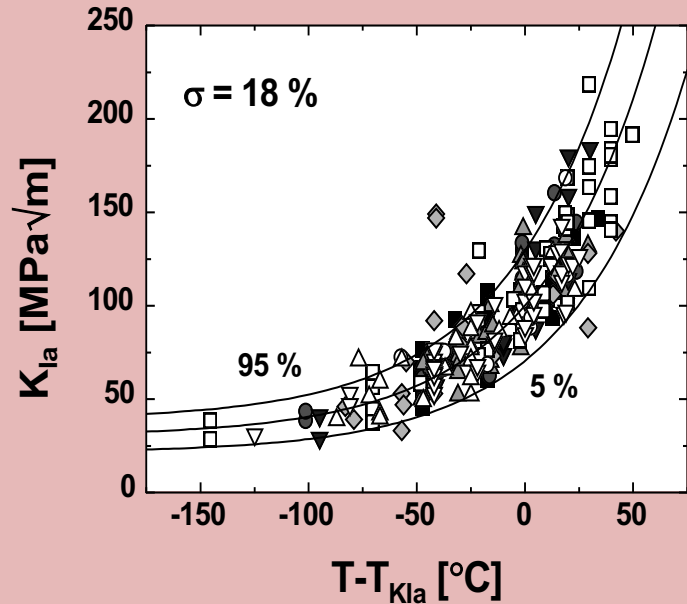


# MC-based Fracture Toughness Models



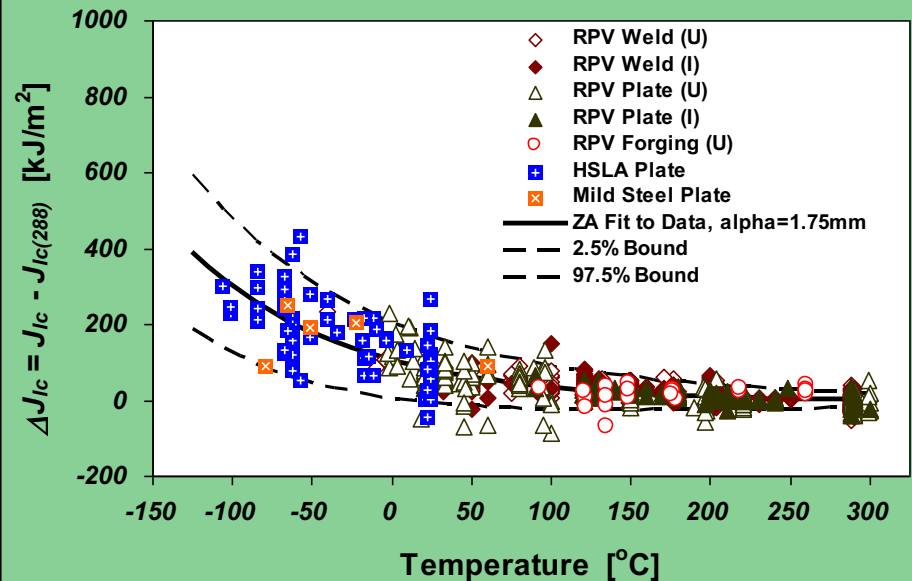
Crack Initiation Toughness:  
 $K_{Jc}$  Master Curve

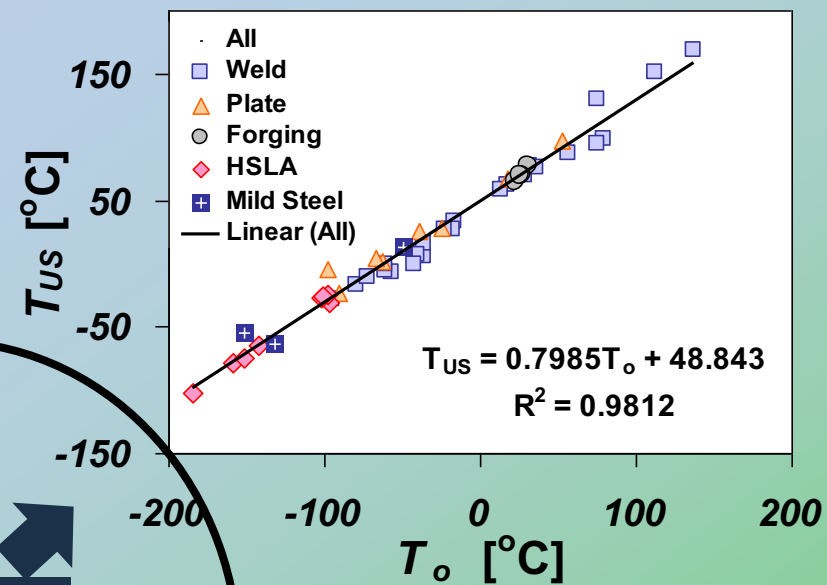
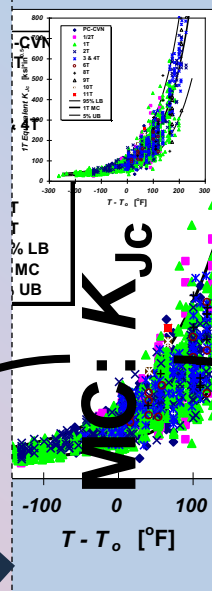
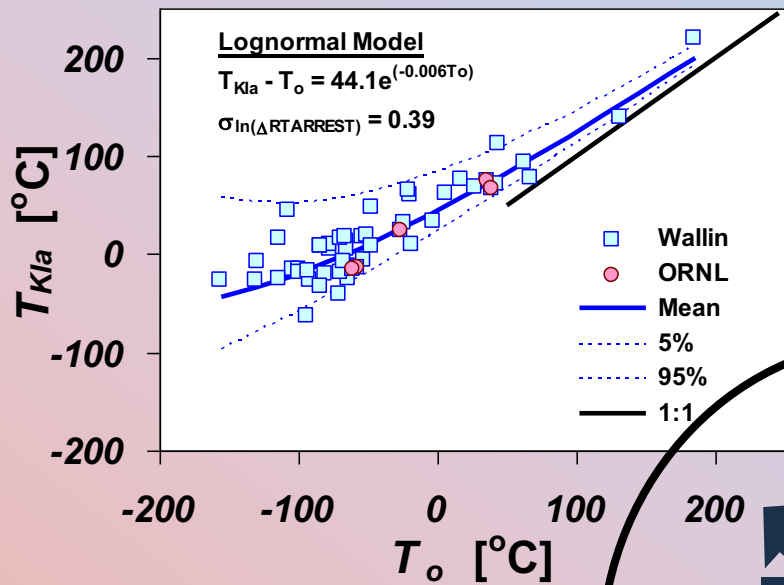
Crack Arrest Toughness:  
 $K_{Ia}$  Master Curve



Primary Toughness Models

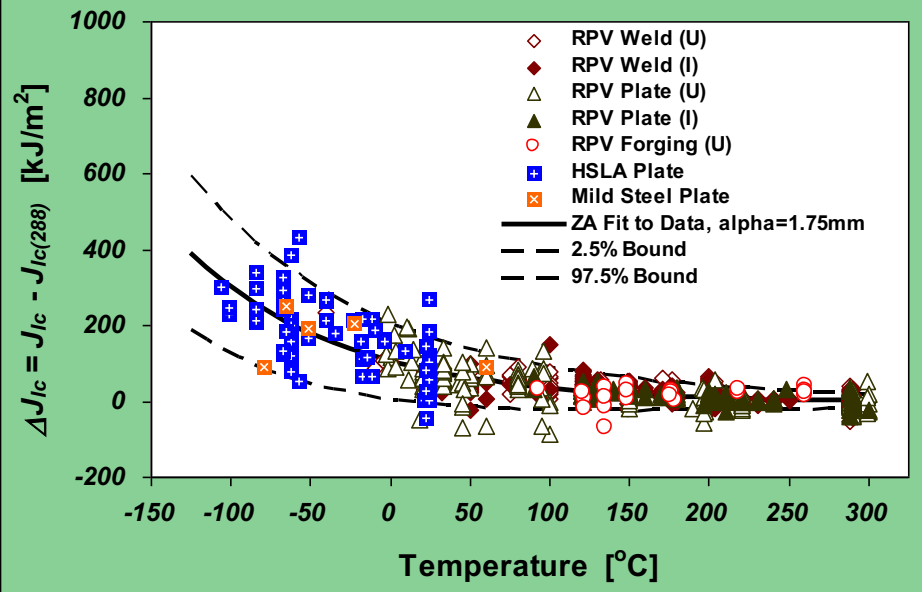
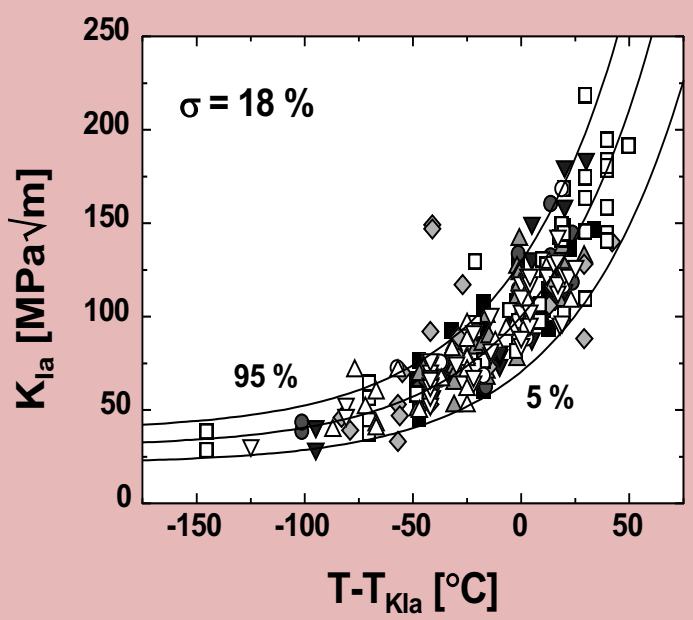
Upper Shelf Toughness:  
 $J_{Ic}$  Master Curve





**Linkage Models**

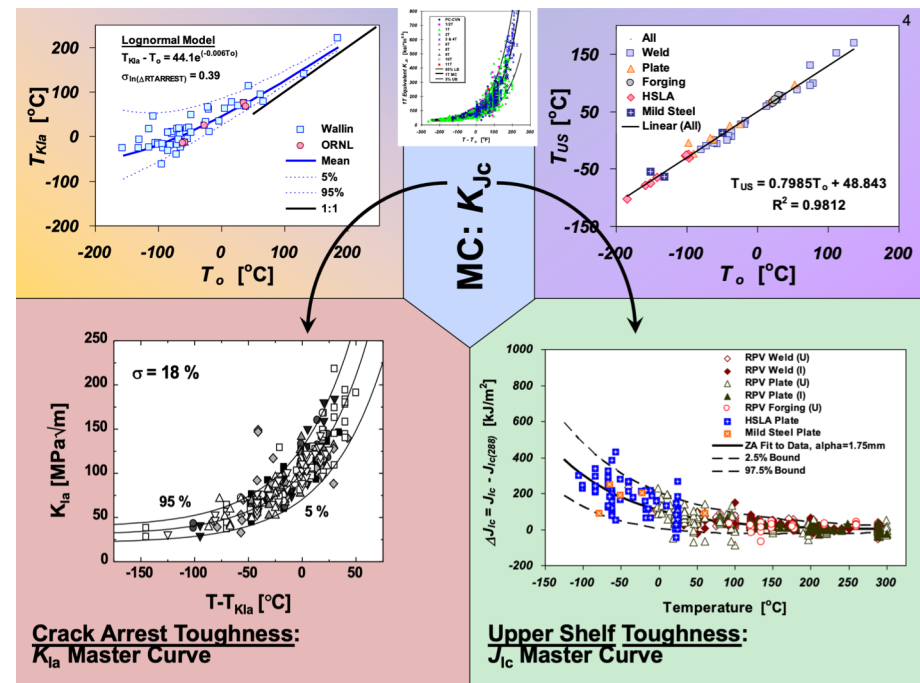
**Crack Arrest Toughness:  $K_{Ia}$  Master Curve**



# Proposed ASME CC N-830-1

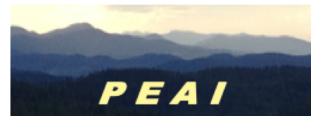
- Recent advancements provide opportunities to adopt best estimate toughness models in the ASME Code
  - Models for toughness vs temperature ( $K_{Jc}$ ,  $K_{Ia}$ ,  $J_{Ic}$ /  $J_{0.1}$ /  $J-R$ )
  - Models to account for systematic linkage between these quantities ( $T_{US}$  and  $T_{KIa}$ )
  - Models provide full statistical distributions, permitting estimation of mean and bounding curves.

- CC N-830-1 proposes a direct-toughness approach
  - Proposes use of MC-based suite of best estimate toughness models as alternative to current Appendix A & K methodology
  - Proposed treatment of uncertainties is consistent with the data used in model development

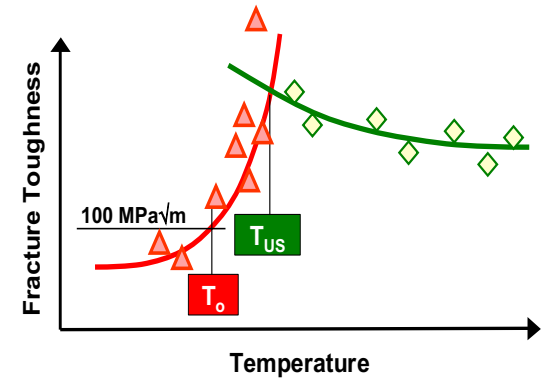


# Uncertainty in N-830-1 Toughness Models

- The toughness metrics ( $K_{Jc}$ ,  $K_{Ia}$ , and  $J_{Ic}/J-R$ ) and the index temperatures ( $T_0$ ,  $T_{Kla}$ ,  $T_{US}$ ) were derived from the same data and thus *reflect the same uncertainties*
  - associated with experimental error (**epistemic**), and
  - material variability (**aleatory**)
- Care was taken to avoid the possibility of '**double counting**,' in the treatment of uncertainties when the models are used together
  - Avoid producing *unrealistic* or overly conservative estimates of fracture toughness.
- The approach adopted in proposed CC N-830-1 is to:
  - **Account** for the experimental error and material variability in the **primary** toughness vs. temperature models
  - **Do not account** for experimental error and material variability in the **linkage models** ( $T_{Kla}$ ,  $T_{US}$ ), which themselves are determined from the  $K_{Jc}$ ,  $K_{Ia}$ , and  $J_{Ic}$ , toughness vs. temperature data.



# Uncertainty Treatment in CC N-830-1



## Primary Model Uncertainty

- **Epistemic Uncertainty Treatment**

- The value of  $T_0$  is adjusted by adding the  $2\sigma$ , where the uncertainty,  $\sigma$ , on  $T_0$  is given in ASTM E1921:

$$\sigma = \sqrt{\left(\frac{\beta^2}{r} + \sigma_{exp}^2\right)} \quad T_{0(ADJ)} = T_0 + 2\sigma$$

- **Aleatory Uncertainty Treatment**

- 5% LB curve taken to describe all toughness values ( $K_{Jc}$ ,  $K_{Ia}$ ,  $J_{Ic}/J_{0.1}/J-R$ )
  - “Bounding toughness curves for a **deterministic analysis** shall be generated from the equations in -4000 by using the values of  $p=0.05$  and  $M_p = 1.64$ . .”
- Sampling on the distributions for each model simulates the uncertainty inherent in the property for use in **probabilistic codes**

Mean values of linkage models ( $T_{US}$  and  $T_{K_{Ia}}$ ) are used

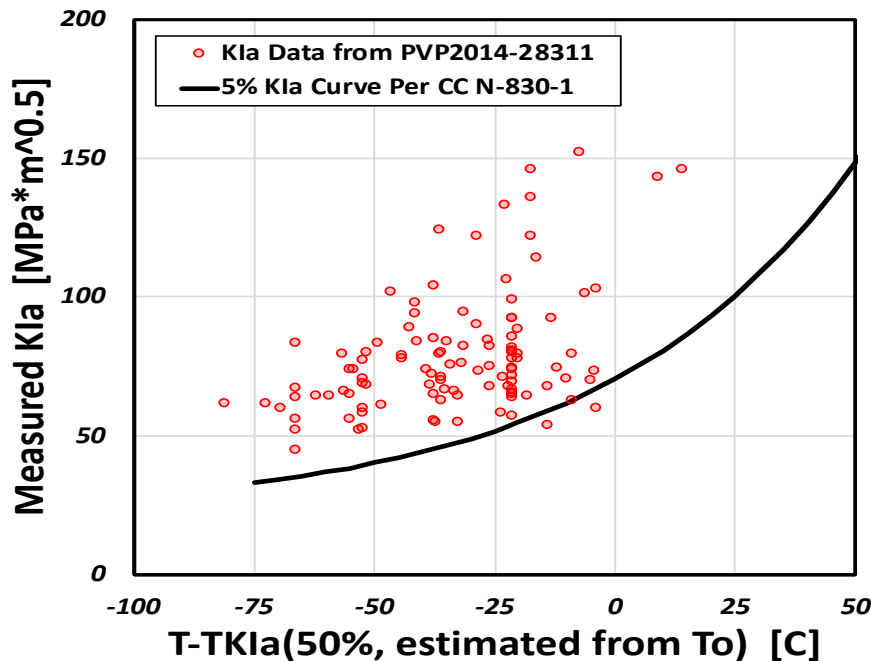


# Validation of Uncertainty Treatment: $T_0$ - $K_{Ia}$

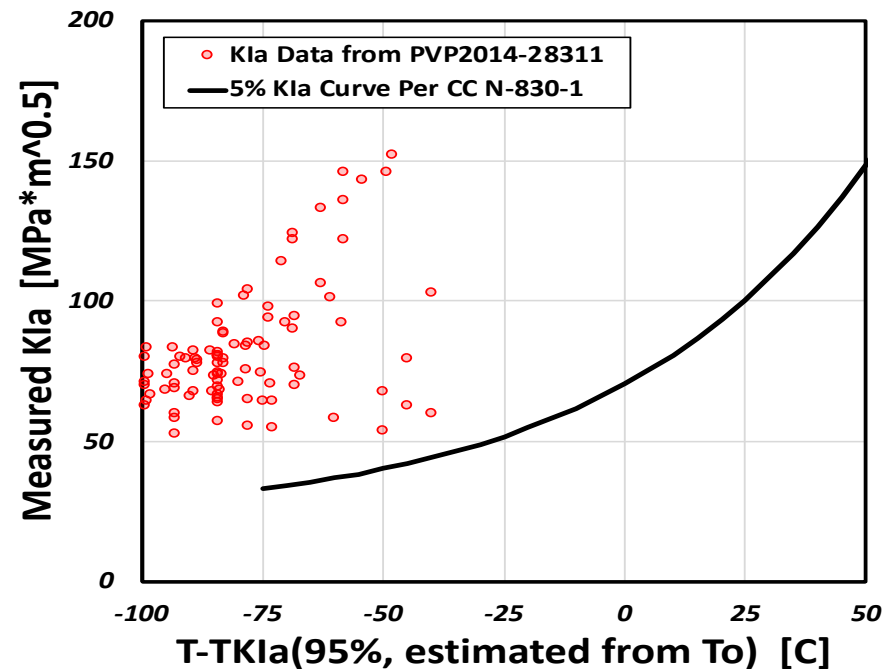
111 measured  $K_{Ia}$  values plotted as a function of  $T-T_{K_{Ia}}$ , where  $T_{K_{Ia}}$  is estimated from measured  $T_0$  values as follows:

$$T_{0(ADJ)} = T_0 + 2\sigma$$

$$T_{K_{Ia}} = T_{0(ADJ)} + 44.97 \exp[-0.00613T_{0(ADJ)}]$$



Approximately 95% bounding is achieved using the mean linkage model:  $T_{0ADJ} - K_{Ia}$



Using the  $T_{0adj}$  AND the  $2\sigma$  lower bound linkage model is WAY over conservative

# Validation of Uncertainty Treatment: $T_0$ - $J_{Ic}$

Four data sets with large number of both  $K_{Jc}$  and  $J_{Ic}$  data are examined to validate the  $T_0 - J_{Ic}$  uncertainty treatment:

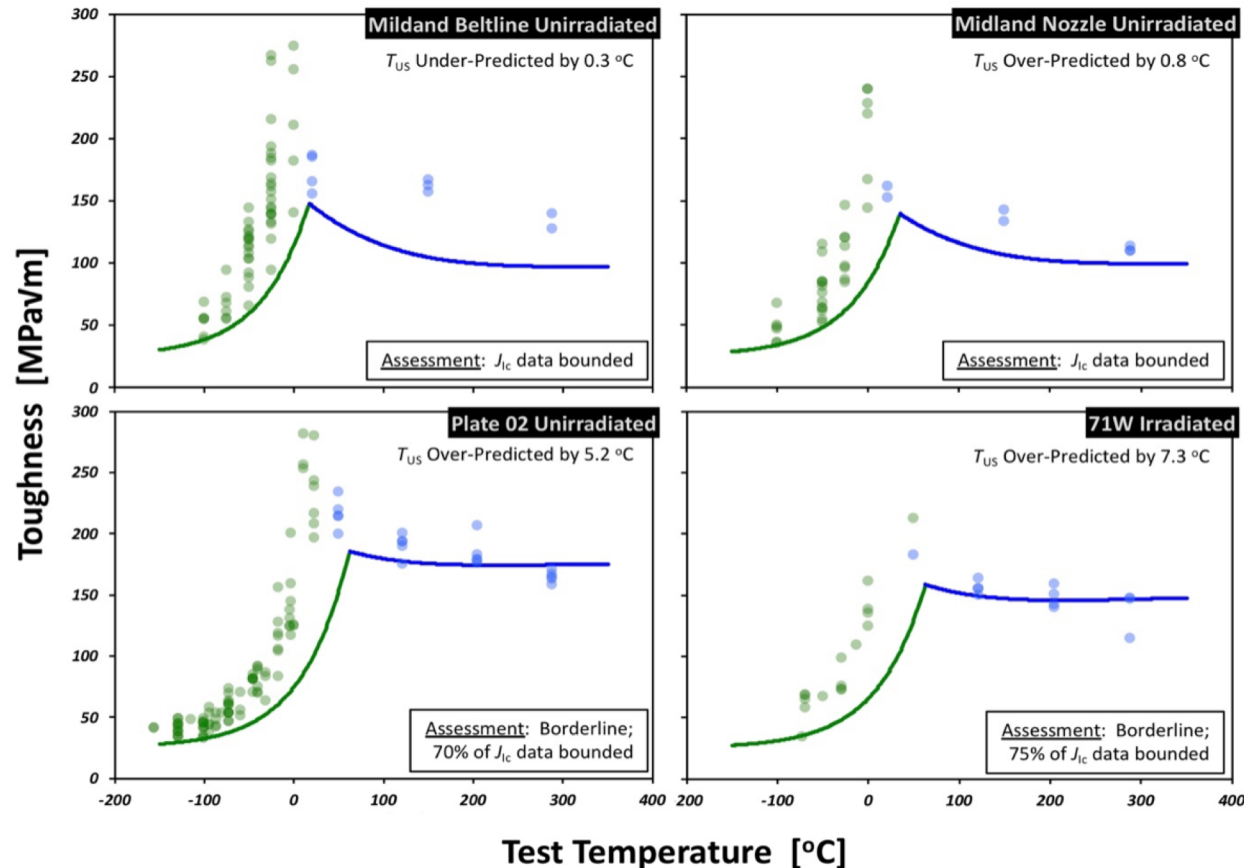
- Midland Beltline (Unirradiated).  $T_{US}$  under-predicted by 0.3 °C.
- Midland Nozzle (Unirradiated).  $T_{US}$  over-predicted by 0.8 °C.
- Plate 02 (Unirradiated).  $T_{US}$  over-predicted by 5.2 °C.
- Weld 71W (Unirradiated).  $T_{US}$  over-predicted by 7.3 °C.

The position of  $K_{Jc}$  and the  $J_{Ic}$  models were determined for each material using:

$$T_{0(ADJ)} = T_0 + 2\sigma$$

$$T_{US} = b_{PF} + 0.84T_{0(ADJ)}$$

Approximately 95% bounding is achieved using the **mean linkage models:  $T_0 - J_{Ic}$**





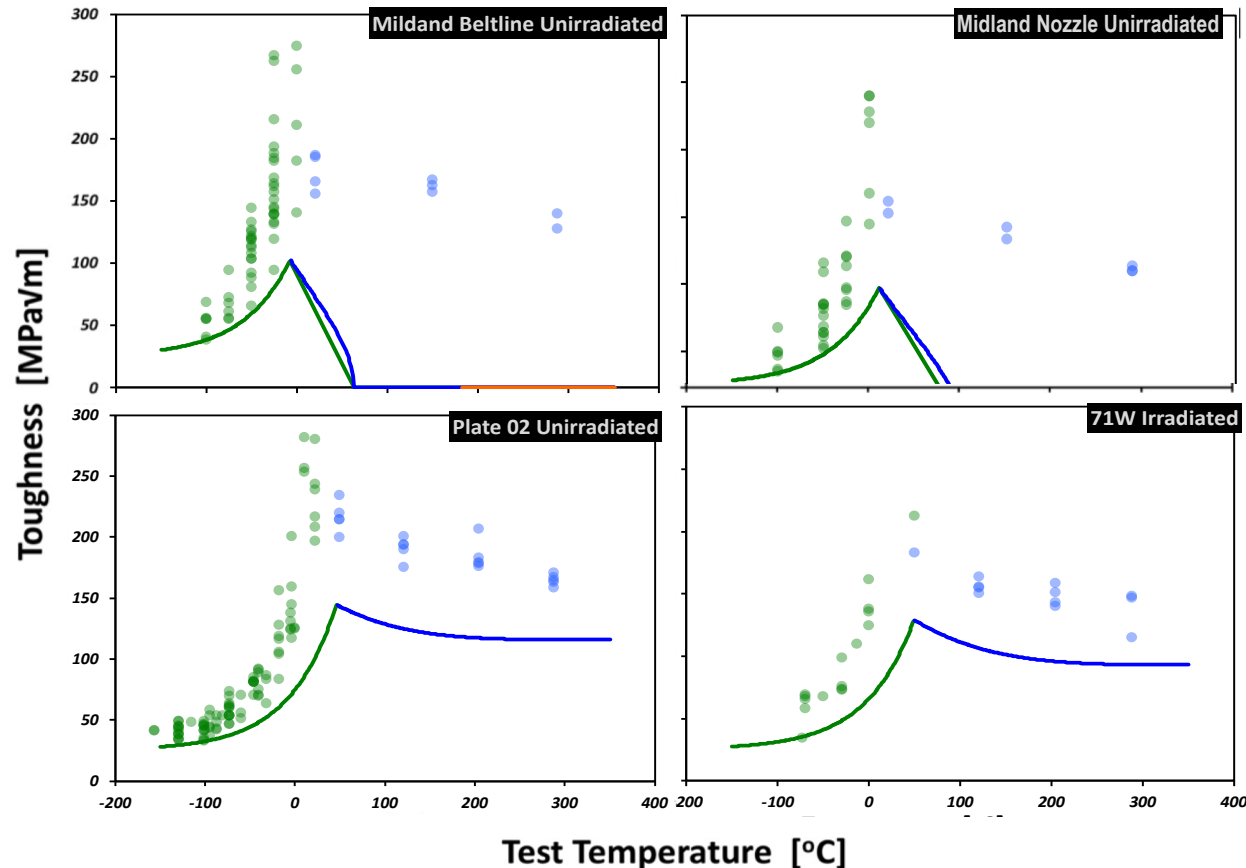
# Validation of Uncertainty Treatment: $T_0$ - $J_{IC}$

Using the same four datasets, the position of  $T_{US}$  and the  $J_{IC}$  model were determined for each material using both  $T_{0(ADJ)}$  and the  $2\sigma$  LB

$$T_{0(ADJ)} = T_0 + 2\sigma$$

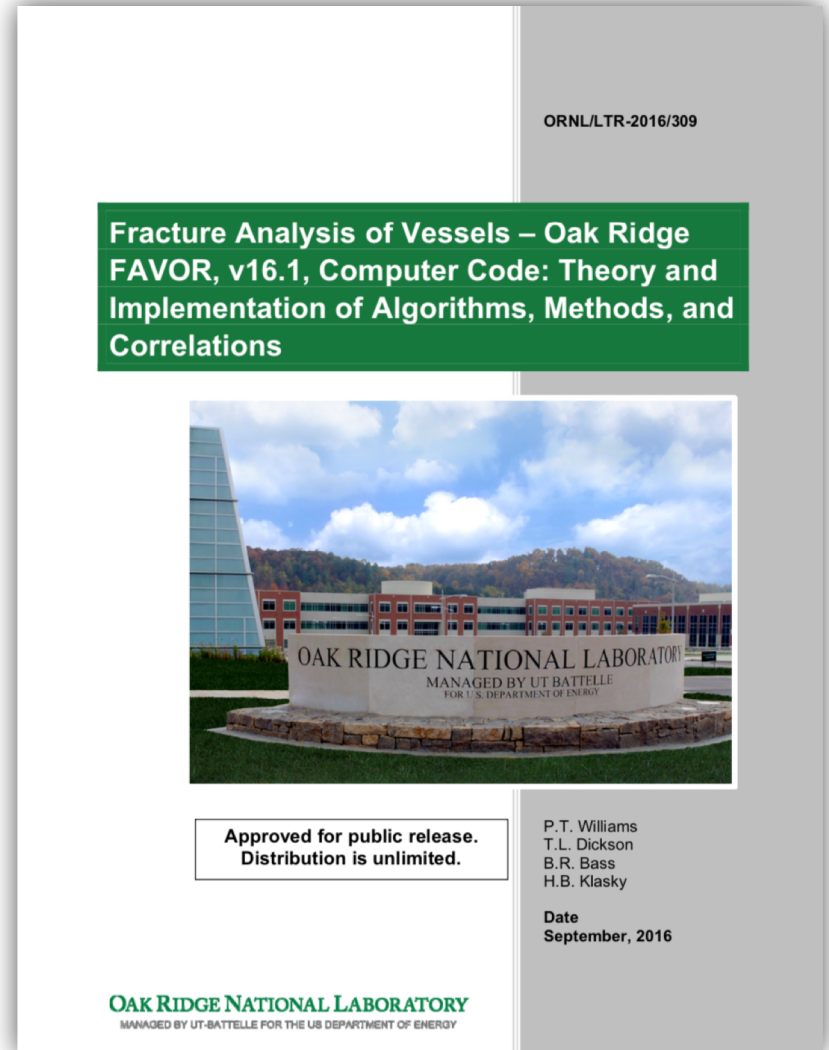
$$T_{US} = b_{PF} + 0.84T_{0(ADJ)} + 2\sigma$$

Bounding is now overly conservative, and in some cases **does not make physical sense** (predictions of  $-J_{IC}$ )

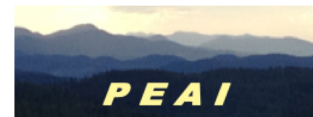


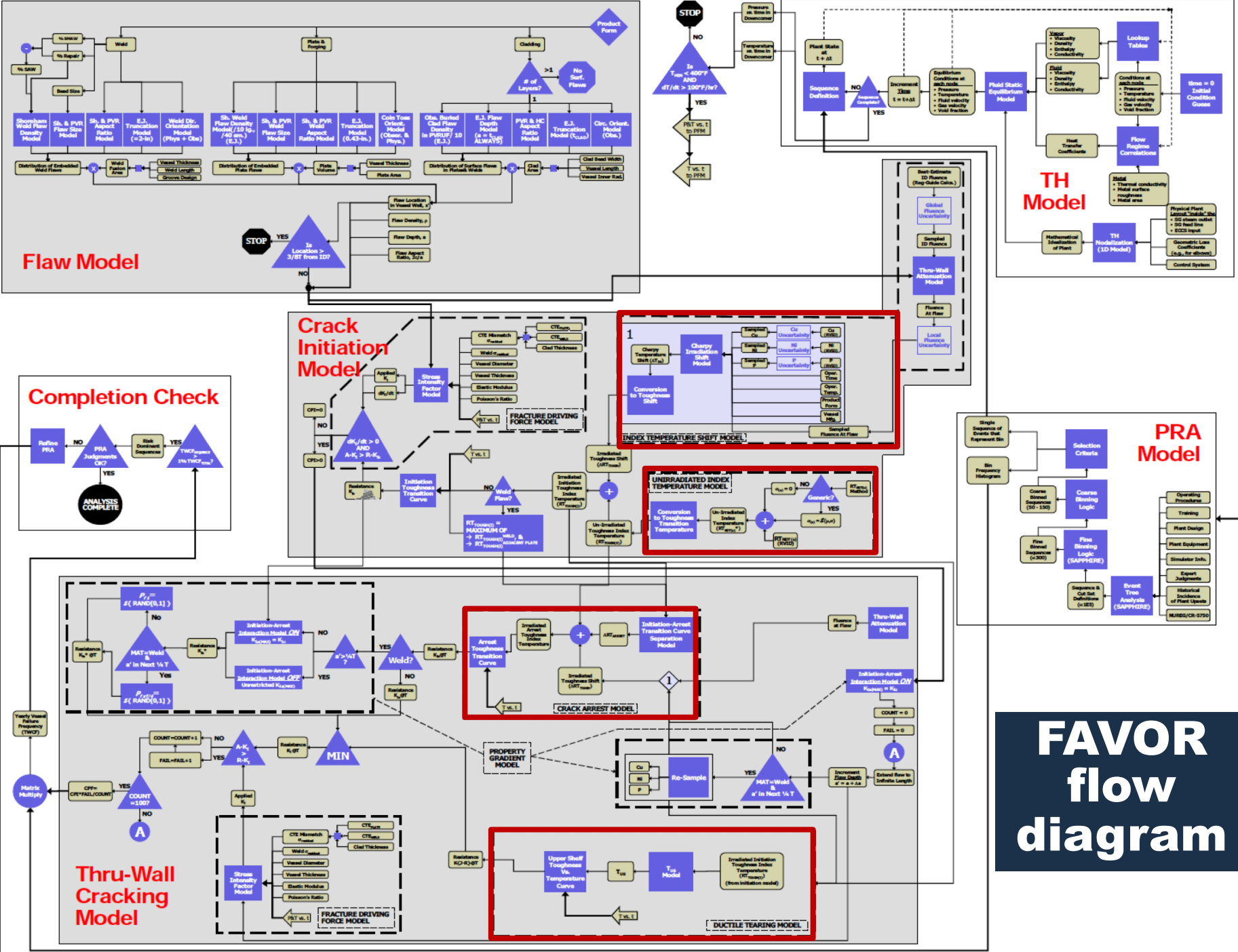
# FAVOR

- Probabilistic code used to assess probability of crack initiation and through-wall cracking probability for RPV steels
- Used by NRC in the development of the alternative PTS rule (10 CFR 50.61a)
  - Extensive internal and external reviews
- Used subsequently by industry and NRC to assess emergent issues
  - BTP 5-3
  - Quasi-laminar flaws
- Used internationally
  - Taiwan
  - Japan
  - Belgium



[ML16273A032, ...A033, ... A034]





**FAVOR flow diagram**

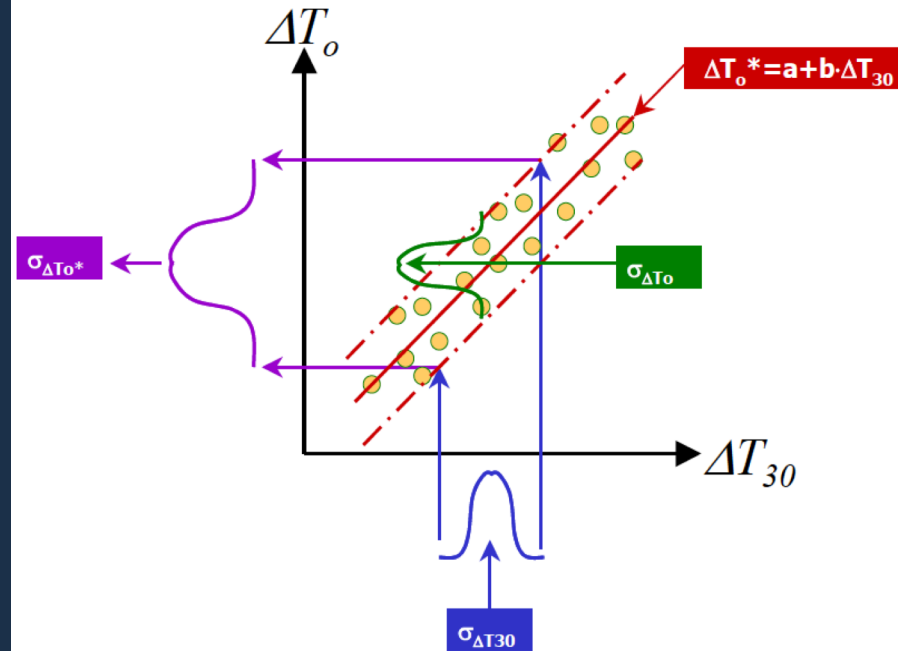
# Uncertainty Treatment in FAVOR

Uncertainties are defined as aleatory or epistemic

- **Epistemic variables** are sampled once for each simulated RPV run and thereafter held fixed. Resampled for next RPV.
- **Aleatory variables** (e.g.  $K_{Ic}$ ) provide the basis for estimating the probability of crack initiation/arrest for each time during a transient

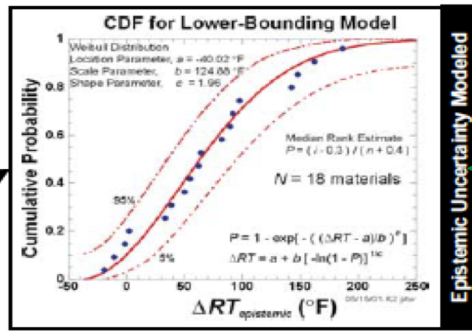
Models are defined as primary or linkage

- Uncertainty is simulated by sampling from values within the defined distribution of each primary model
- Uncertainty simulation in the linkage models is mixed
  - Mean values are used for  $T_{US}$ ,  $\Delta T_{30}$
  - Uncertainty accounted for in  $T_{KIa}$  ( $\Delta RT_{arrest}$ )



# FAVOR Crack Initiation Model

Uncertainty NOT accounted for in linkage models

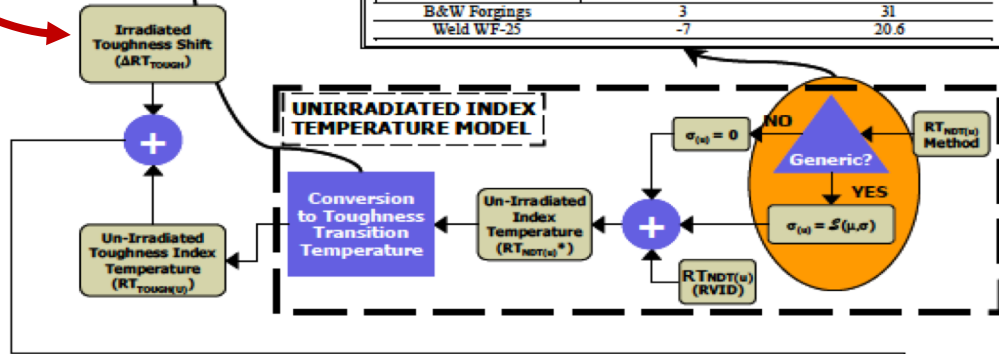


Epistemic Uncertainty Modeled

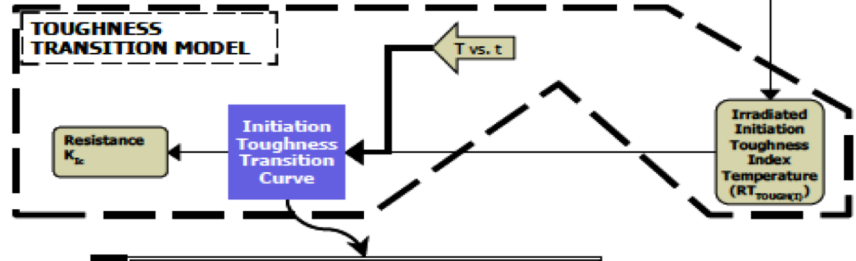
model sampled to simulate uncertainty in  $RT_{NDT}$

$$RT_{LB} = RT_{NDT} - \Delta RT_{epistemic}$$

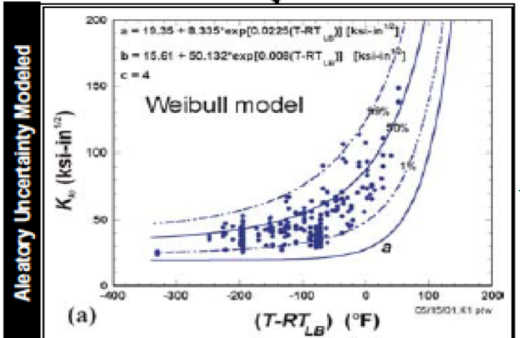
Material Group	Generic $RT_{NDT}$ Value (mean) [°F]	Standard Deviation [°F]
CE Welds	-56	17
B&W Welds	-5	17
B&W Plates	1	26.9
B&W Forgings	3	31
Weld WF-25	-7	20.6



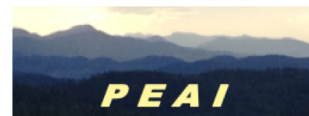
START



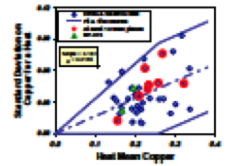
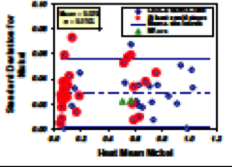
model sampled to simulate uncertainty in  $K_{Ic}$

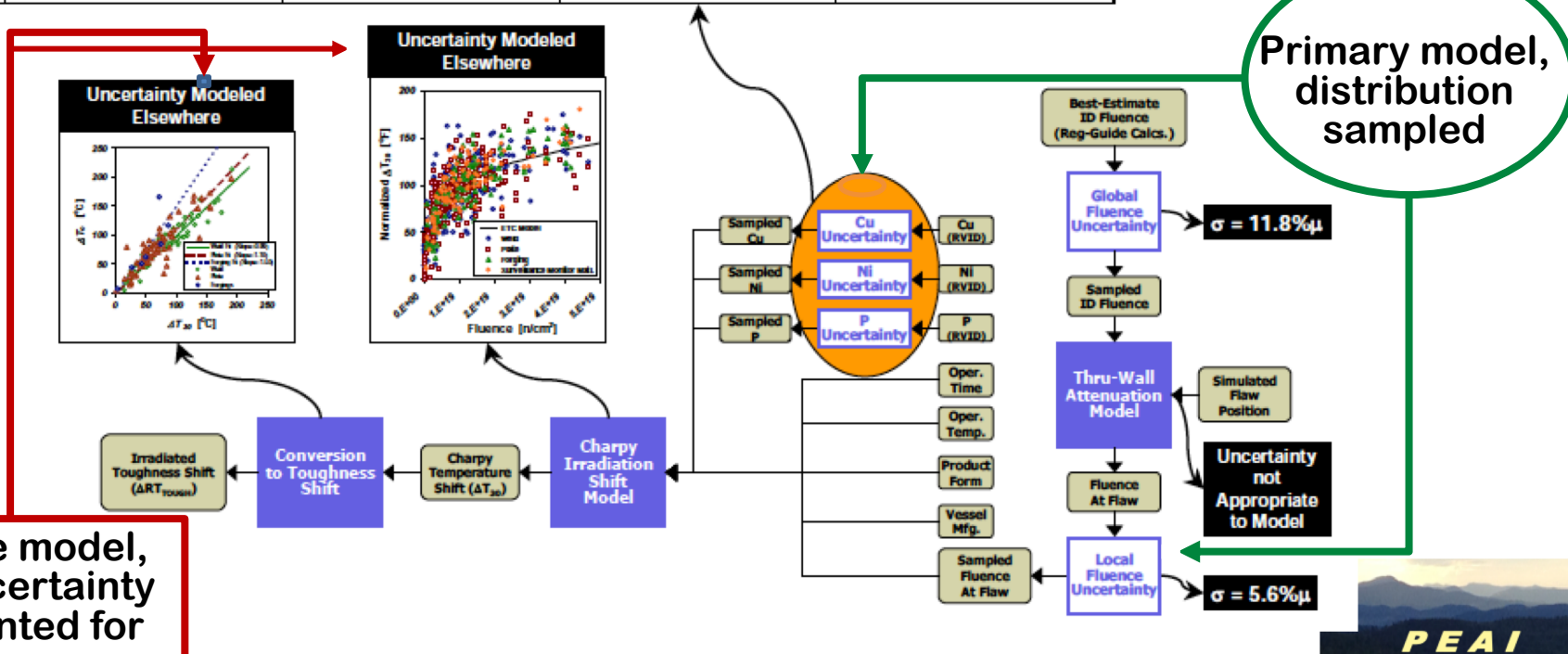


Aleatory Uncertainty Modeled



# FAVOR Embrittlement Model

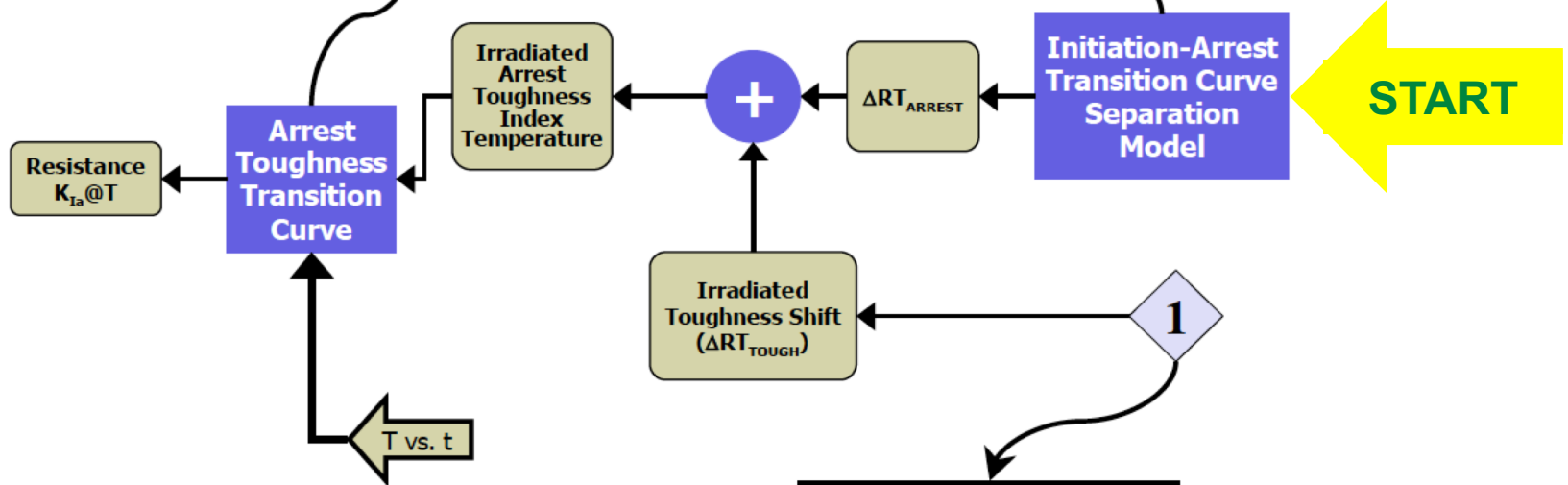
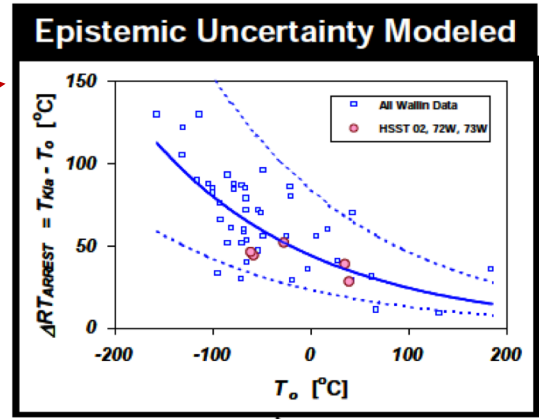
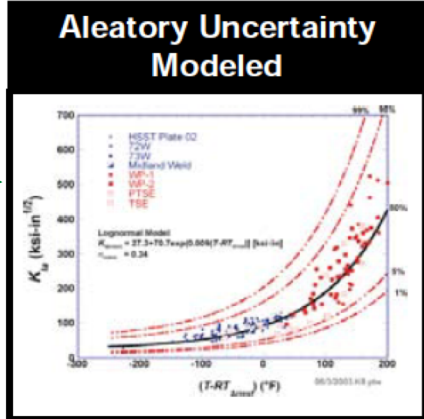
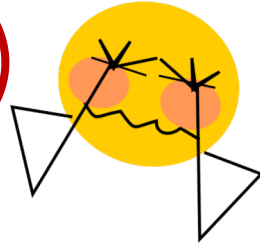
	Weld $\sigma$		Plate & Forging $\sigma$	
	Sub-Region	Local	Sub-Region	Local
<b>Cu</b>		0.0131	0.0073	0.0035
<b>Ni</b>		0.0119	0.0244	0.0124
<b>P</b>	0.0013	0.0008	0.0013	0.0008



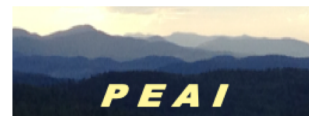
# FAVOR Crack Arrest Model

model sampled to simulate uncertainty in  $K_{Ia}$

Uncertainty IS accounted for in linkage models



Embrittlement Model, See Fig, 3-41



# Summary and Conclusions

- Information flow through a code controls the cumulative impact of various sources of uncertainty on failure probabilities.
- Sub-models must be linked together appropriately to ensure that there is no distortion of the flow of information
- To avoid double counting of uncertainty contribution from various sources
  - Account for uncertainty in primary models (derived directly from data), but
  - Do NOT account for uncertainties in linkage models derived from primary models
    - Can result in excessive conservatism, or
    - Can result in non-physically realistic predictions of model outcomes
- Perhaps there is an analogy to complex human relationships?

