Excellence through Collaboration

Estimation of Threshold Parameter and Its Uncertainty Using Multi-Variable Modeling Framework for Response Variable with Binary Experimental Outcomes

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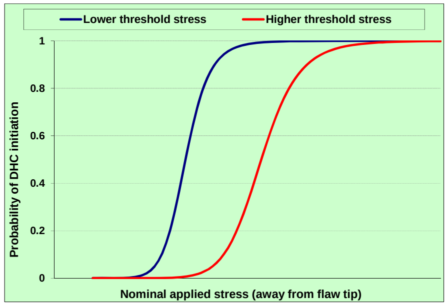
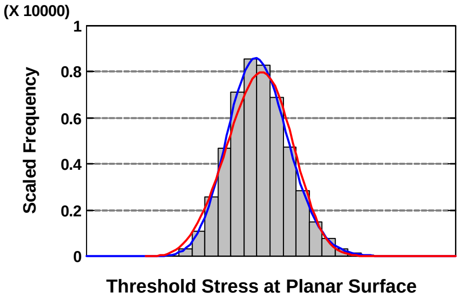
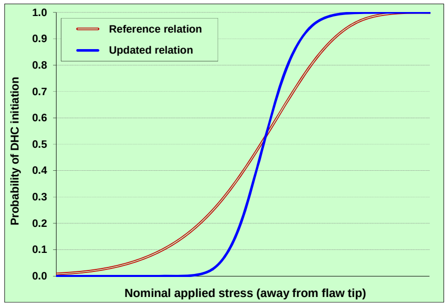
Outline







- **Background**
- **Approach**
- **Modeling Framework**
- **Analysis Results**
- **Summary**

Background

Pressure tubes in CANDU reactors

CANDU® reactor core: 380 to 480 fuel channels
with nuclear fuel and pressurized heavy water coolant

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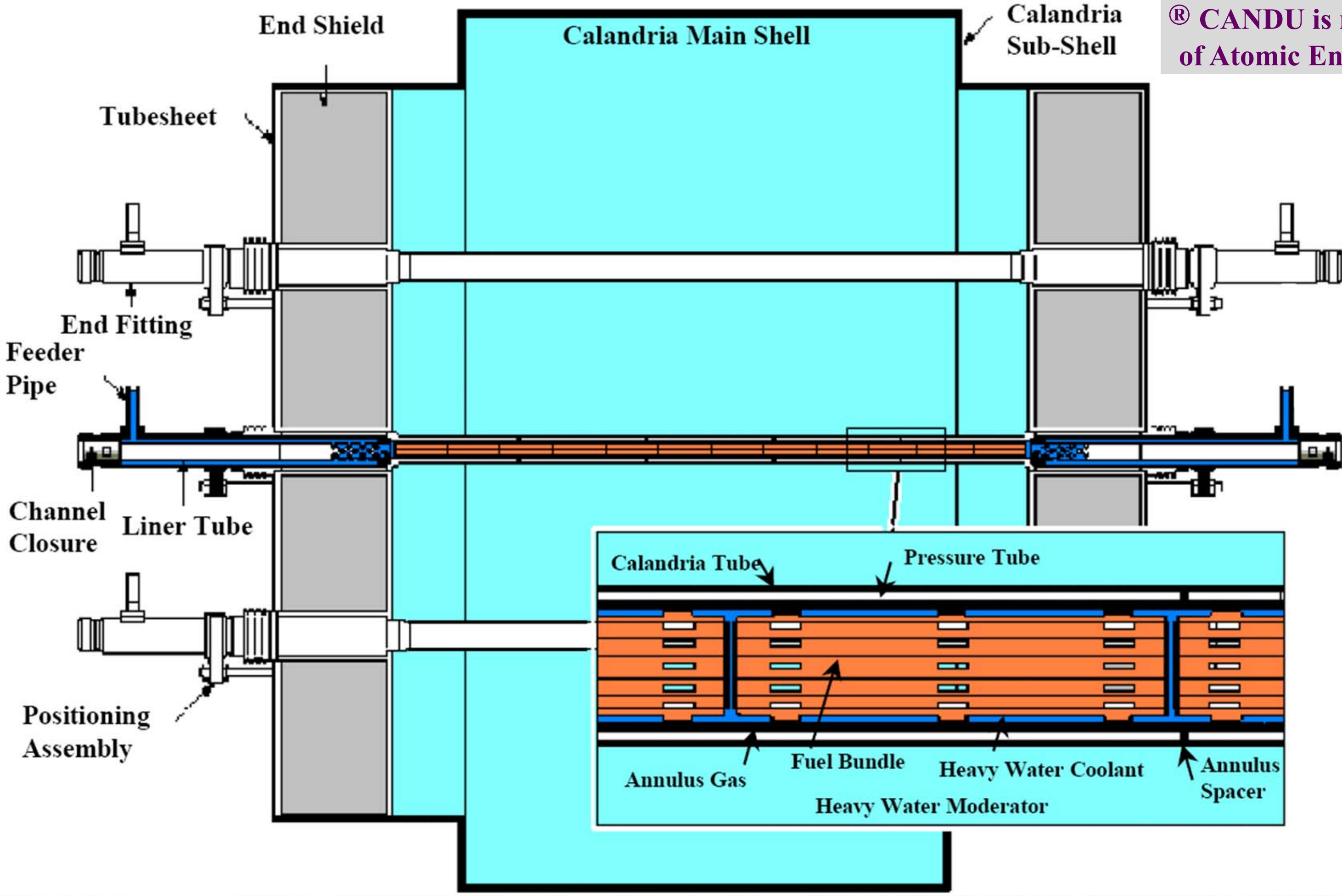
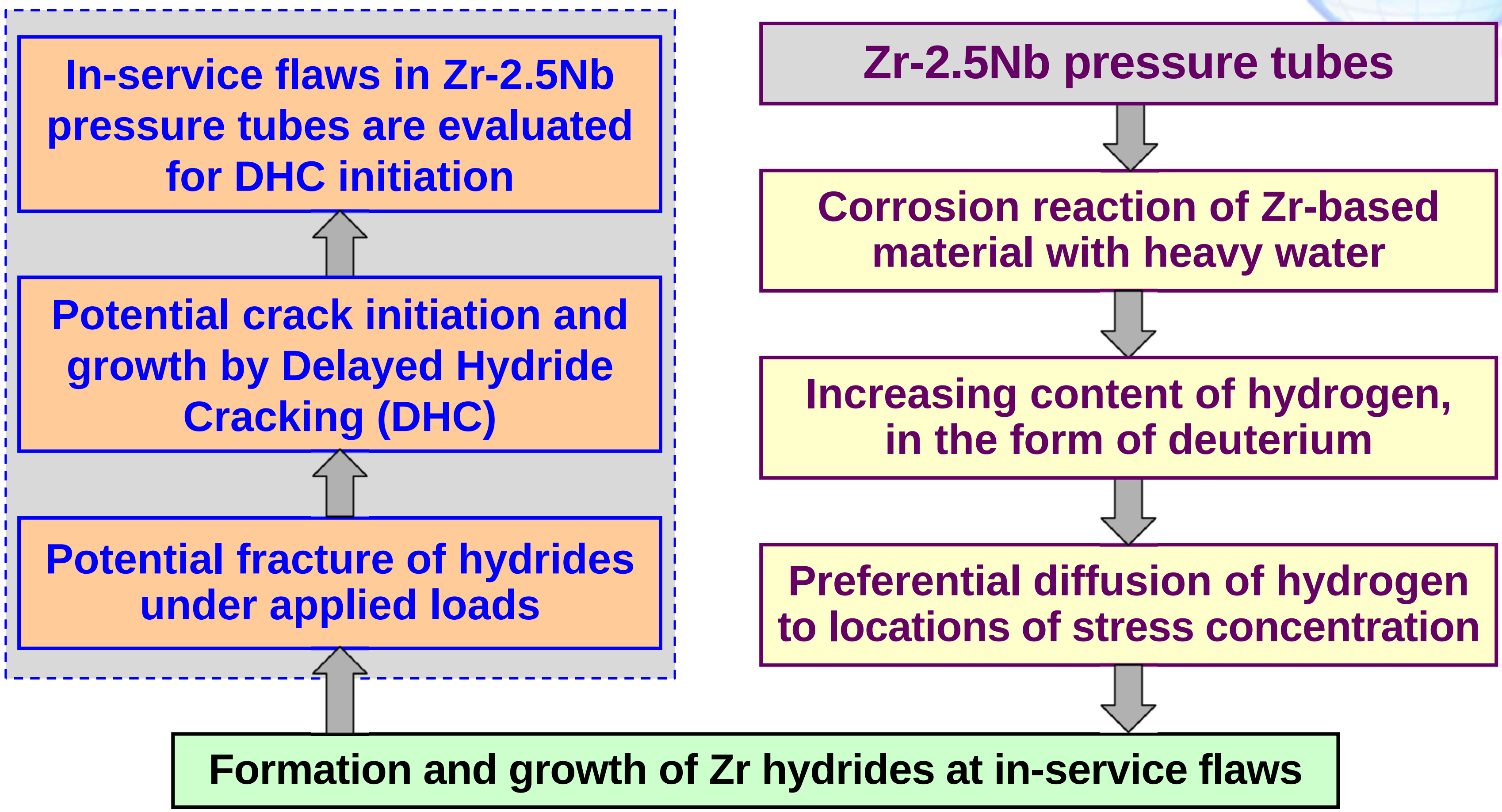


Background

Delayed hydride cracking in CANDU pressure tubes



Background

Threshold stress for DHC initiation

DHC initiation occurs when applied stress exceeds threshold stress

applied stress

$$\sigma_{a,tip} > \sigma_{th,tip}$$

$$\sigma_{a,n} > \sigma_{th,n}$$

threshold stress for DHC initiation

Model based on strip-yield process-zone approach is used to predict threshold stress for DHC initiation (CSA Standard N285.8)

threshold stress intensity factor for DHC initiation from cracks

yield stress

flaw depth

pressure tube wall thickness

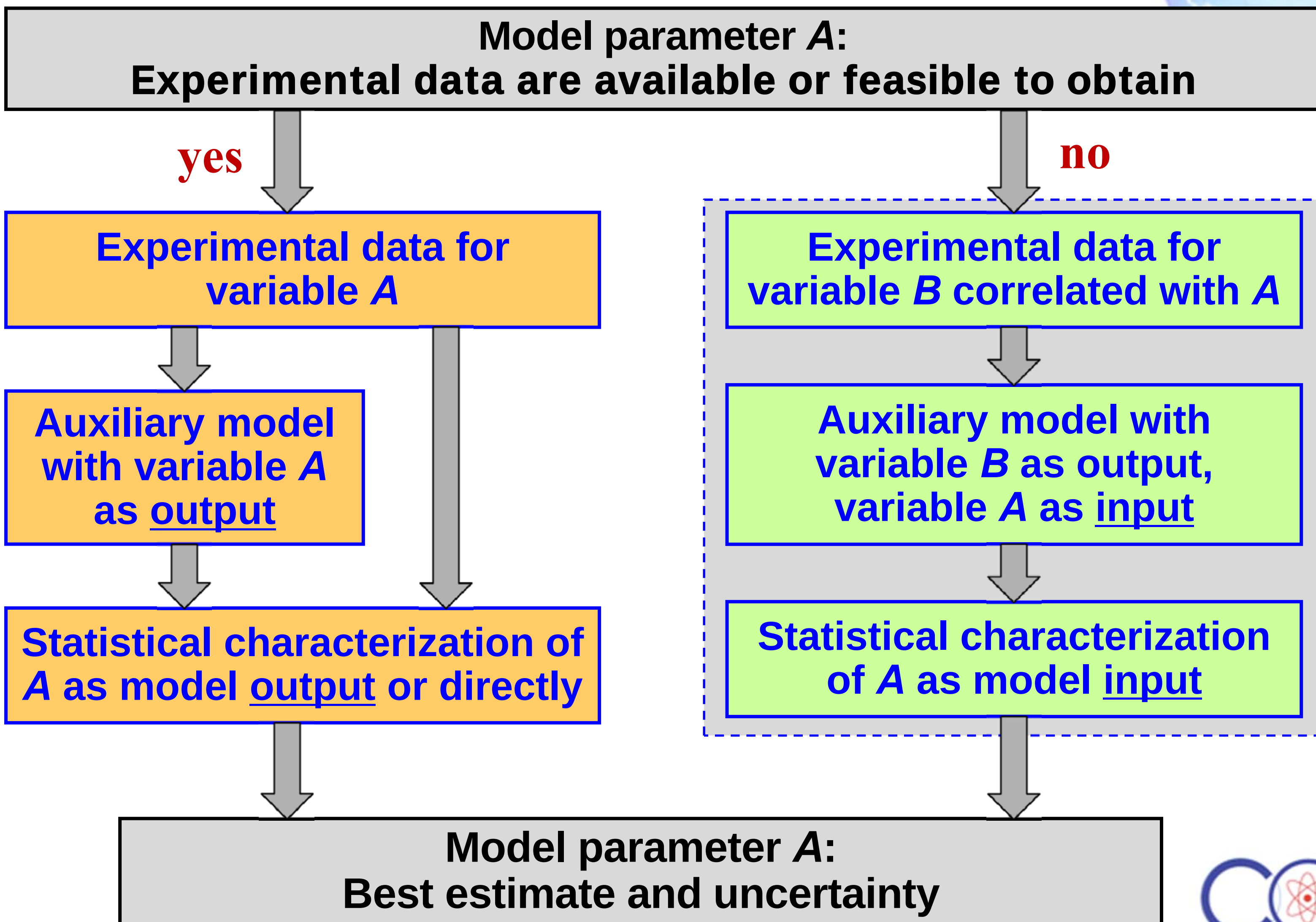
stress concentration factor

$$\sigma_{th,tip} = \sigma_{th}^{(ps)}\left[1 + \Psi\left(k_t, \frac{K_{IH}}{\sigma_{th}^{(ps)}\sqrt{w}}, \frac{\sigma_{yield}}{\sigma_{th}^{(ps)}}, \frac{a}{w}\right)\right]$$

threshold stress for DHC initiation at planar surface

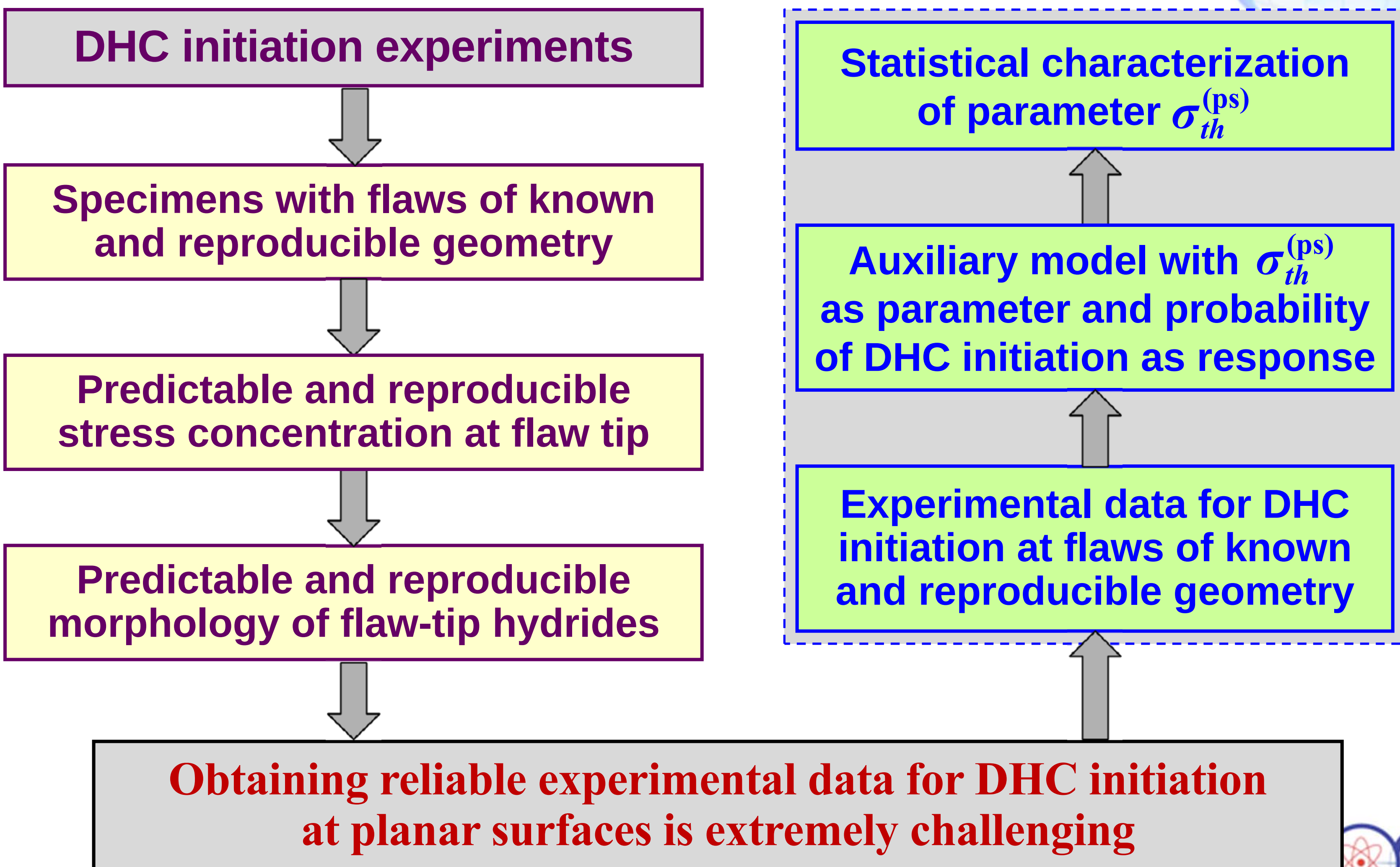
Approach

Parameter estimation from experimental data



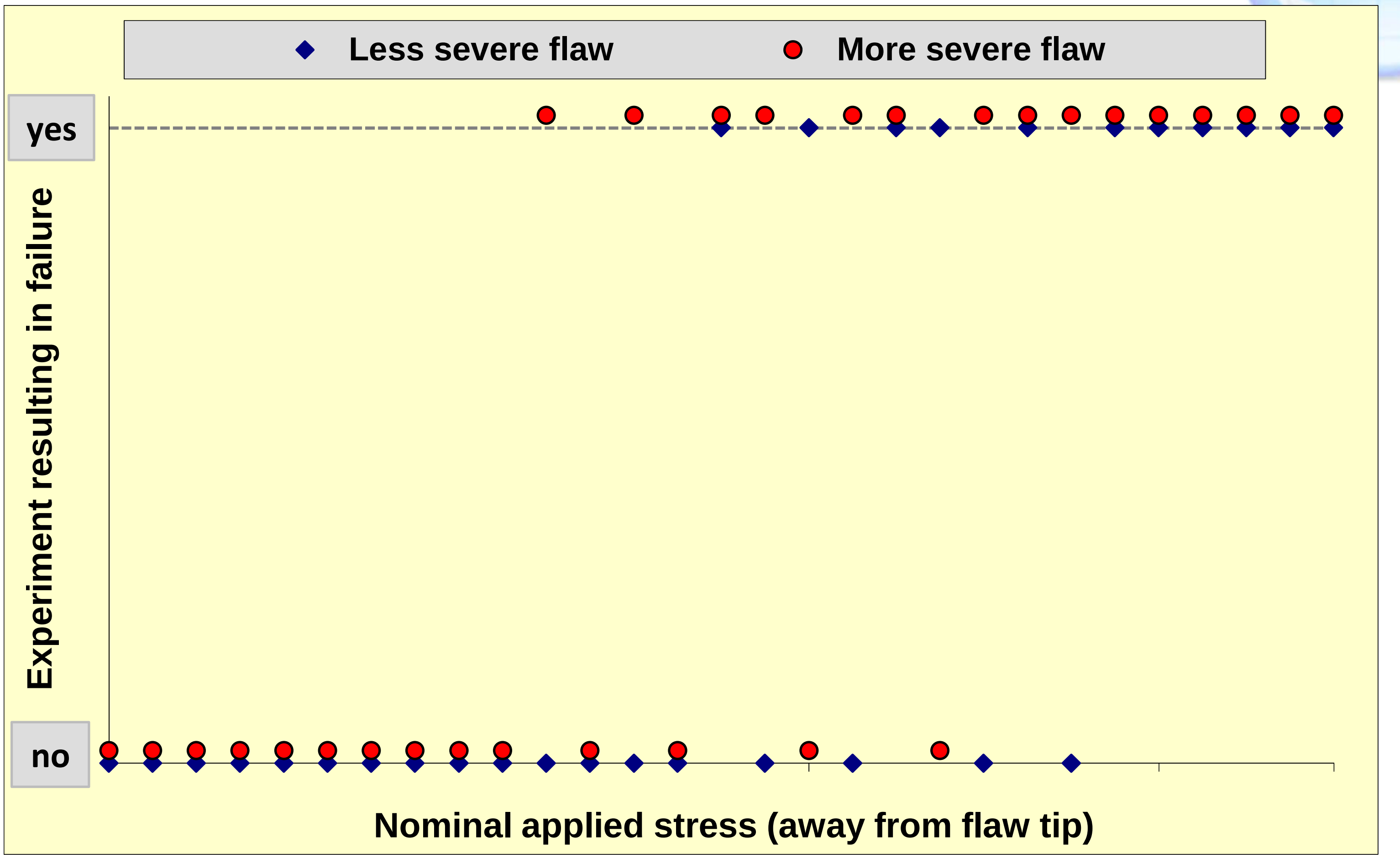
Approach

Overview of approach to $\sigma_{th}^{(ps)}$ characterization



Approach

Results of DHC initiation experiments



If multiple results are available at a given test condition, proportion of failures can be used

Approach

Auxiliary model for probability of DHC initiation

a) Flaw geometry

b) Material resistance to DHC initiation, including $\sigma_{th}^{(ps)}$

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Threshold stress for DHC initiation

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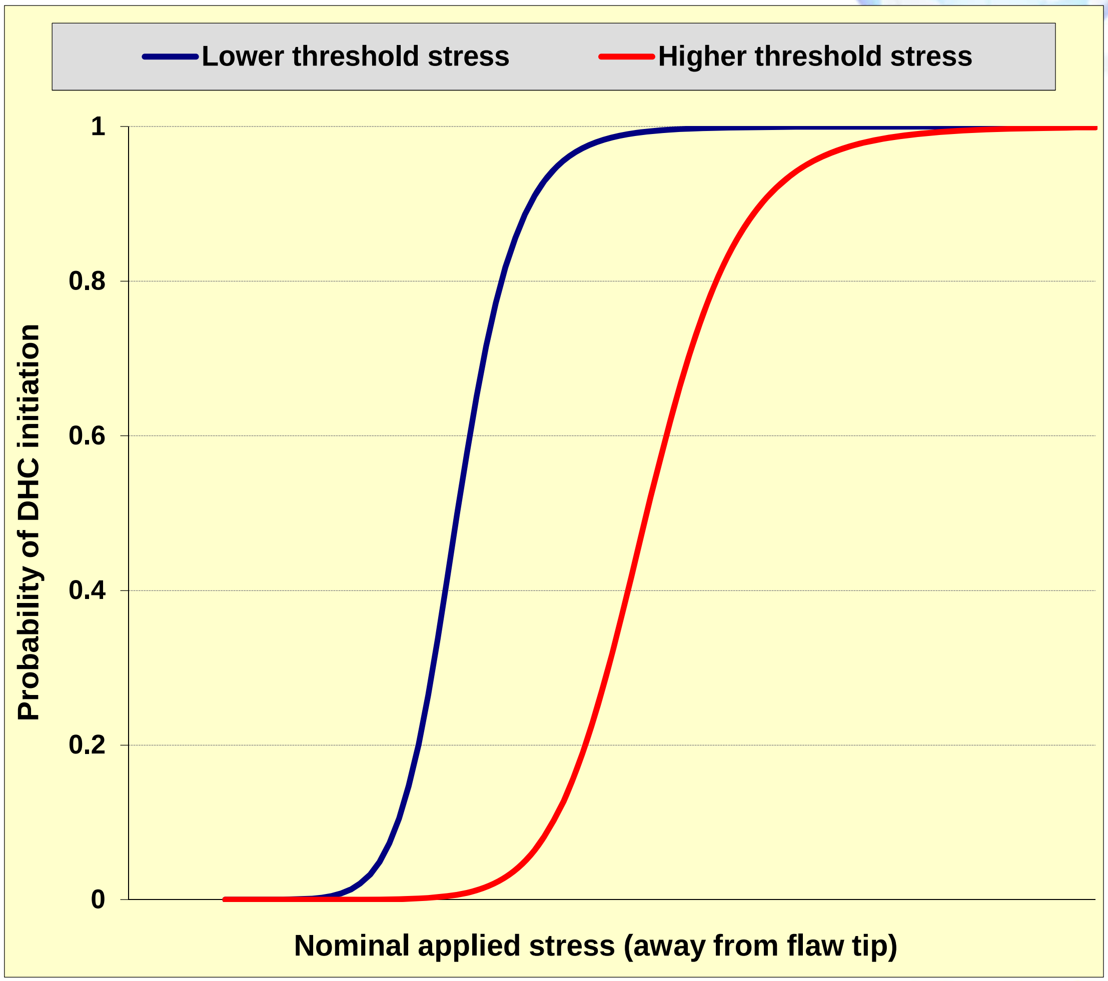
Position of zero-to-one transition in probability of DHC initiation along applied stress axis



Approach

Logistic regression analysis

expected probability of DHC initiation

residual uncertainty

$$L(\Pi_{CI}) = L(\tilde{\Pi}_{CI}) + \varepsilon_{\Pi}$$

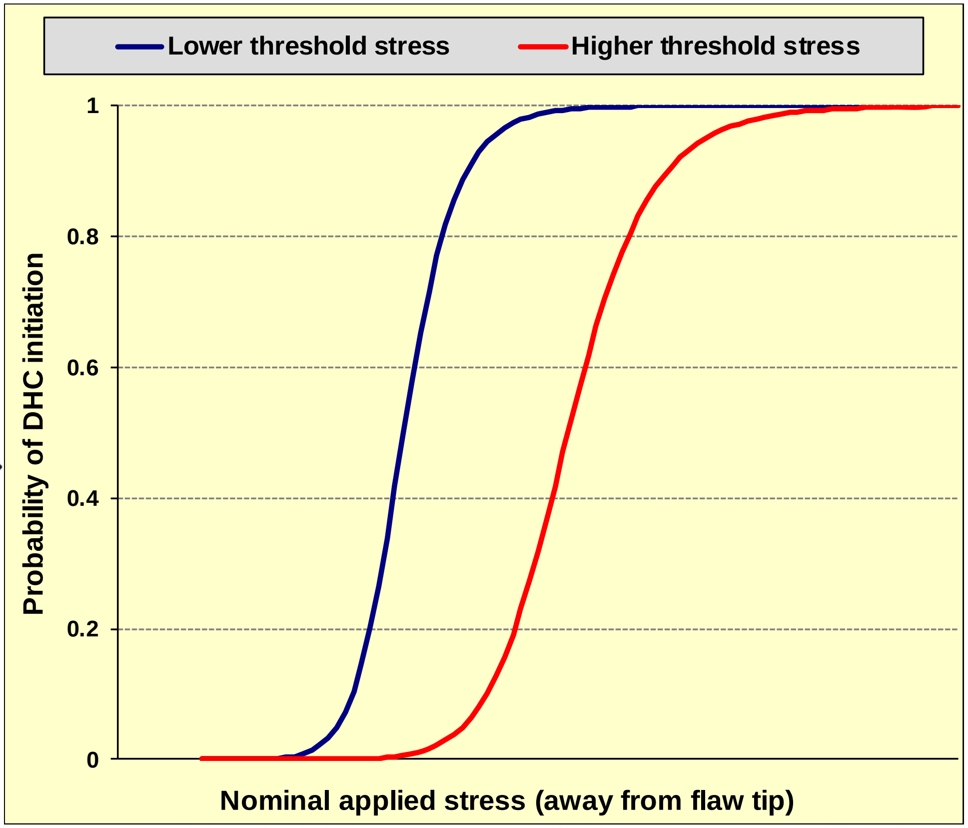
observed probability of DHC initiation

$$L(\Pi_{CI}) = \ln\left(\frac{\pi_{CI}}{1-\pi_{CI}}\right)$$

π_{CI}: proportion of failures

$$L(\tilde{\Pi}_{CI}) = B_{\sigma} \ln\left(\frac{\sigma_a}{\sigma_{th}}\right)$$

B_{σ}: empirical parameter



Modeling Framework

Generalized predictor in auxiliary model

$$\sigma_{a,tip} = k_t\, \sigma_{a,n}$$

- flaw-tip applied stress: $\sigma_{a,tip}$
- stress concentration factor: k_t
- nominal applied stress: $\sigma_{a,n}$

$$L(\tilde{\Pi}_{CI}) = B_\sigma \ln\left(\frac{\sigma_{a,tip}}{\sigma_{th,tip}}\right) = B_\sigma \ln\left(\frac{\sigma_{a,n}}{\sigma_{th,n}}\right)$$

$$\sigma_{th,tip} = \frac{\sigma_{th}^{(ps)}}{1-\lambda}\left(Z_{CI}\, F\left[\frac{a}{w}\right]\frac{K_{IH}}{\sigma_{th}^{(ps)}\sqrt{w}}\right)^{\lambda}$$

- corrective function: Z_{CI}
- geometry factor: F

$$\sigma_{th,tip} = \sigma_{th}^{(ps)}\left[1 + \Psi\left(\lambda, \frac{K_{IH}}{\sigma_{th}^{(ps)}\sqrt{w}}, \frac{\sigma_{yield}}{\sigma_{th}^{(ps)}}, \frac{a}{w}\right)\right]$$

Small effect of plasticity outside of process zone: simplified relation for threshold stress

flaw severity factor

$$0 \le \lambda = \frac{k_t - 1}{k_t} < 1$$

Modeling Framework

Relation for threshold stress at planar surface

$$Z_{CI} = Z_{CI}^{(0)} \neq f\left(\frac{a}{w}, \frac{K_{IH}}{\sigma_0\sqrt{w}}\right)$$

$$L(\tilde{\Pi}_{CI}) = B_0 + B_\lambda(1-\lambda) + B_\sigma\left[\ln\left(\frac{\sigma_{a,n}}{\sigma_0}\right) - \lambda\ln\left(F\left[\frac{a}{w}\right]\frac{K_{IH}}{\sigma_0\sqrt{w}}\right)\right]$$

Empirical parameters B_0, B_λ, B_σ: correlated random variables

$$B_0 = -B_\sigma \ln(Z_{CI}^{(0)}); \qquad B_\lambda = -B_\sigma\left[\ln\left(\frac{\sigma_{th}^{(ps)}}{\sigma_0}\right) - \ln(Z_{CI}^{(0)})\right]$$

normalizing constant

$$\sigma_{th}^{(ps)} = \sigma_0 \exp\left(-\frac{B_0 + B_\lambda}{B_\sigma}\right)$$

Modeling Framework

Threshold stress at planar surface: varying Z_{CI}

$$Z_{CI} = Z\left(\frac{a}{w}, \frac{K_{IH}}{\sigma_0\sqrt{w}}\right) = Z_{CI}^{(0)}\left(F\left[\frac{a}{w}\right]\right)^{Z_{CI}^{(F)}}\left(\frac{K_{IH}}{\sigma_0\sqrt{w}}\right)^{Z_{CI}^{(K)}}$$

$$L(\tilde{\Pi}_{CI}) = B_0 + B_\lambda(1-\lambda) + B_\sigma \ln\left(\frac{\sigma_{a,n}}{\sigma_0}\right) + B_F\lambda\ln\left(F\left[\frac{a}{w}\right]\right) + B_K\lambda\ln\left(\frac{K_{IH}}{\sigma_0\sqrt{w}}\right)$$

$$B_F = -B_\sigma[1 + Z_{CI}^{(F)}]; \qquad B_K = -B_\sigma[1 + Z_{CI}^{(K)}]$$

$$B_0 = -B_\sigma \ln(Z_{CI}^{(0)}); \qquad B_\lambda = -B_\sigma\left[\ln\left(\frac{\sigma_{th}^{(\mathrm{ps})}}{\sigma_0}\right) - \ln(Z_{CI}^{(0)})\right]$$

$$\sigma_{th}^{(\mathrm{ps})} = \sigma_0 \exp\left(-\frac{B_0 + B_\lambda}{B_\sigma}\right)$$

Modeling Framework

Potential correlation between $\sigma_{th}^{(\mathrm{ps})}$ and K_{IH}

$$\sigma_{th}^{(\mathrm{ps})}(K_{IH}) = Q_0 \left(\frac{K_{IH}}{\sigma_0 \sqrt{w}} \right)^{Q_1}$$

$$L(\tilde{\Pi}_{CI}) = B_0 + B_\lambda (1-\lambda) + B_\sigma \left[\ln\left(\frac{\sigma_{a,n}}{\sigma_0} \right) - \lambda \ln\left(F\left[\frac{a}{w}\right] \frac{K_{IH}}{\sigma_0\sqrt{w}} \right) \right] + B_Q (1-\lambda) \ln\left(\frac{K_{IH}}{\sigma_0\sqrt{w}} \right)$$

$$B_0 = -B_\sigma \ln(Z_{CI}^{(0)}); \quad B_\lambda = -B_\sigma \left[\ln\left(\frac{Q_0}{\sigma_0} \right) - \ln(Z_{CI}^{(0)}) \right]; \quad B_Q = -B_\sigma Q_1$$

$$Q_0 = \sigma_0 \exp\left(-\frac{B_0 + B_\lambda}{B_\sigma} \right); \qquad Q_1 = -\frac{B_Q}{B_\sigma}$$

Modeling Framework

Relation for stress $\sigma_{th}^{(ps)}$: smooth surface data

$$\lambda = \frac{k_t - 1}{k_t} = 0$$

$$L(\tilde{\Pi}_{CI}) = B_\sigma^{ss} \ln\left(\frac{\sigma_{a,n}}{\sigma_{th}^{(ps)}}\right) = B_0^{ss} + B_\sigma^{ss} \ln\left(\frac{\sigma_{a,n}}{\sigma_0}\right)$$

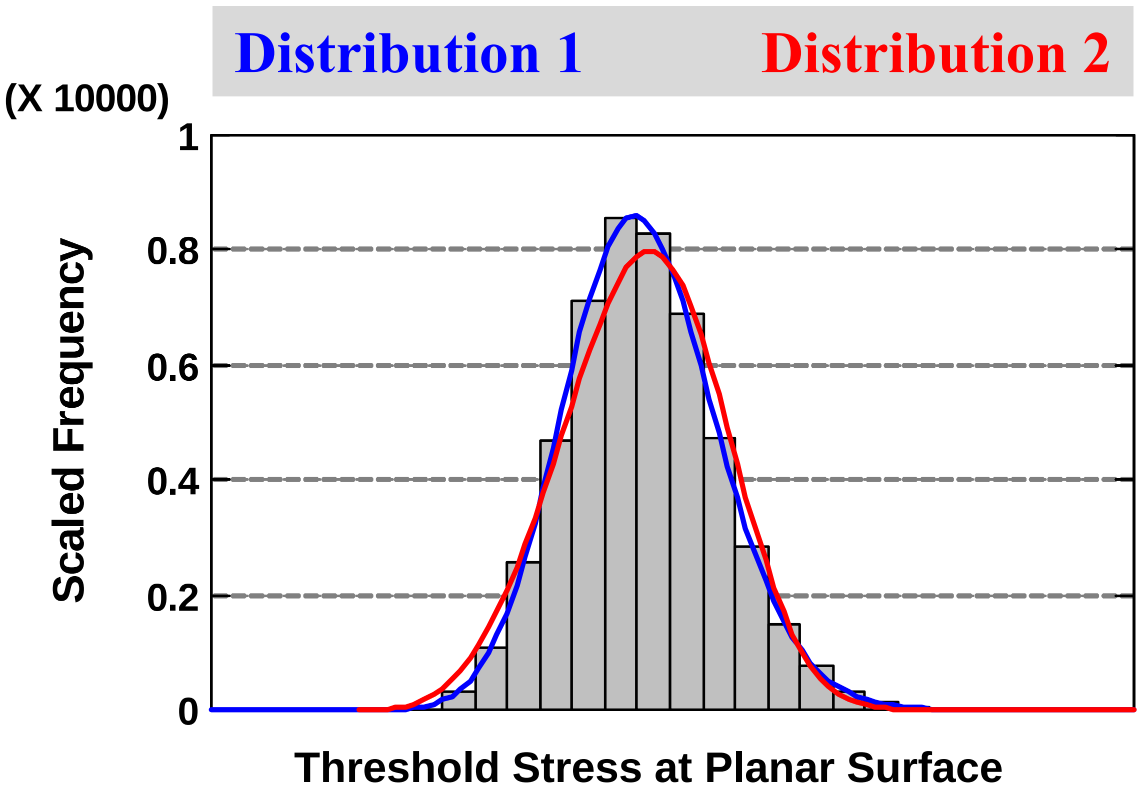
Empirical parameters B_0^{ss}, B_σ^{ss}: correlated random variables

$$B_0^{ss} = -B_\sigma^{ss} \ln\left(\frac{\sigma_{th}^{(ps)}}{\sigma_0}\right)$$

$$\sigma_{th}^{(ps)} = \sigma_0 \exp\left(-\frac{B_0^{ss}}{B_\sigma^{ss}}\right)$$

Analysis Results

Best estimate of $\sigma_{th}^{(ps)}$ and its uncertainty



simulated triplets of B_0, B_λ, B_σ

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$$\sigma_{th}^{(ps)} = \sigma_0 \exp\left(-\frac{B_0 + B_\lambda}{B_\sigma}\right)$$

↓

probability density function of $\sigma_{th}^{(ps)}$

Analysis Results

Reference and updated probabilistic relations

Reference relation

$$\sigma_{th}^{(ps)} = \mathrm{W}_2(\mu_W, \beta_W)$$

two-parameter Weibull distribution

Scale parameter μ_W

Shape parameter β_W

Median value $= \mu_W \left[\ln(2)\right]^{1/\beta_W}$

Updated relation

$$\sigma_{th}^{(ps)} = \mathrm{L}_3(\mu_L, \beta_L, \delta_L)$$

three-parameter log-normal distribution

Location parameter μ_L

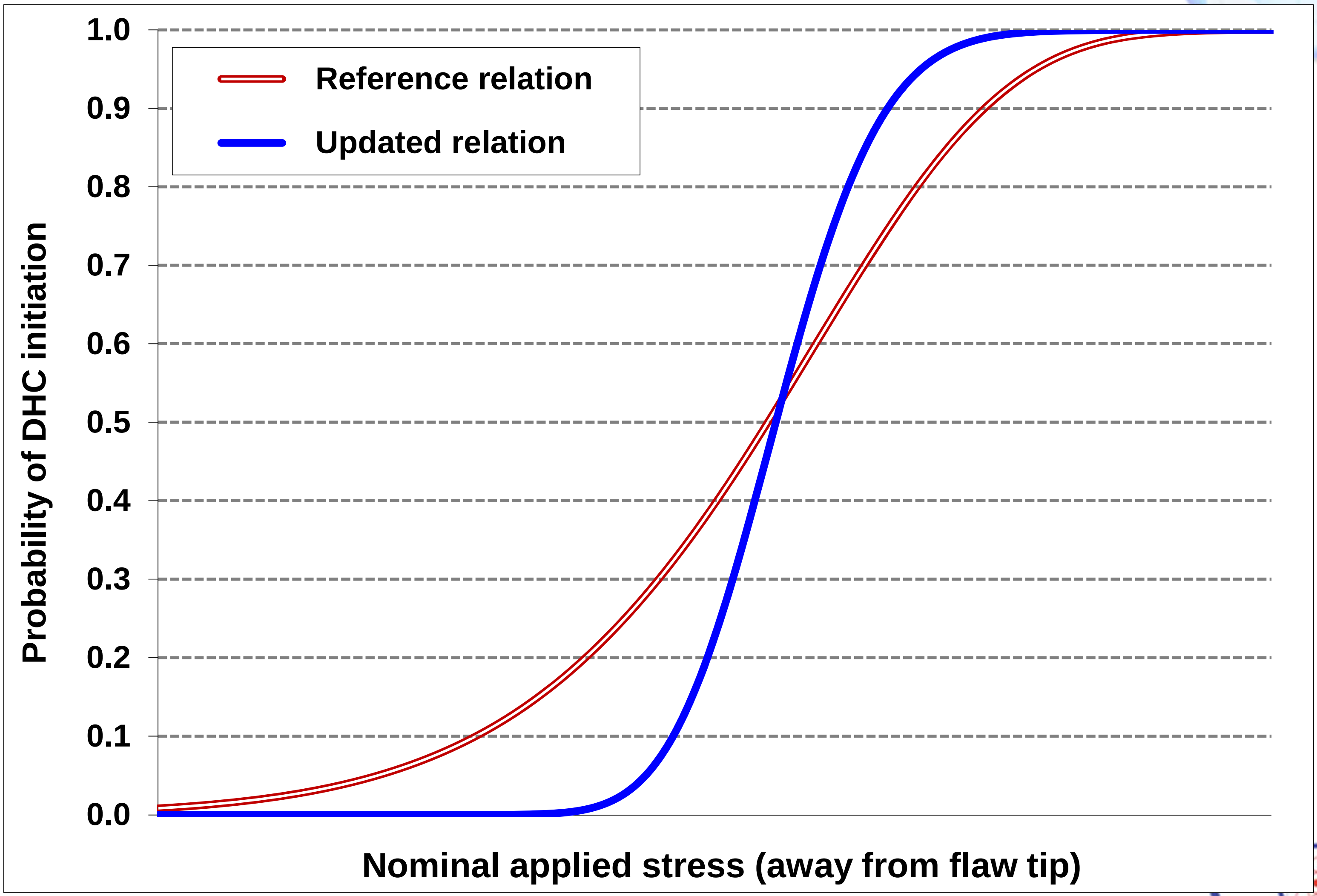
Scale parameter β_L

Lower threshold δ_L

Median value $= \delta_L + \exp(\mu_L)$

Analysis Results

Reference and updated probabilistic relations



Summary

Obtaining reliable experimental data for initiation of delayed hydride cracking (DHC) at planar surfaces is extremely challenging

Auxiliary multi-variable modeling framework has been developed to characterize threshold stress for DHC initiation at planar surfaces, $\sigma_{th}^{(ps)}$, as a random variable:

- **Modeling framework is based on closed-form representation of threshold stress for DHC initiation at flaws, as developed from strip-yield process-zone approach to modeling crack initiation**
- **Higher probability of DHC initiation is inferred for**
  - **flaws of greater severity**
  - **lower material resistance to DHC initiation**
- **$\sigma_{th}^{(ps)}$ is one of parameters in developed framework**

Summary

Developed modeling framework can be applied to:

- **Statistically assessing binary outcomes of DHC initiation experiments performed on specimens containing flaws of varying severity**
- **Characterizing threshold stress for DHC initiation at planar surfaces as a non-deterministic input parameter for relevant probabilistic evaluations**
- **Investigating likely correlation between threshold stress for DHC initiation at planar surfaces and K_{IH} (threshold stress intensity factor for DHC initiation from a crack)**

Application of developed framework is under consideration

Acknowledgements

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- **Support provided by CANDU Owners Group**

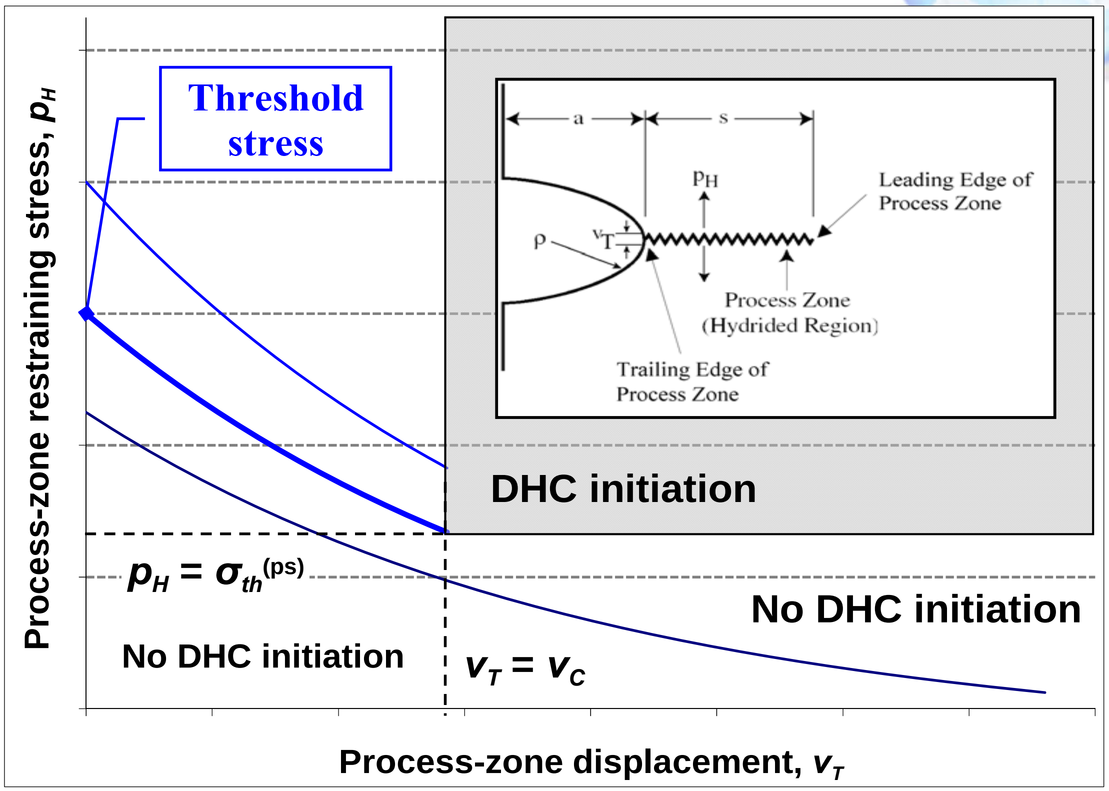
Thank you

Background

Process-zone approach to DHC initiation

DHC initiation does not occur until v_T reaches v_C, a critical flaw-tip displacement

DHC initiation does not occur as long as restraining stress p_H remains below $\sigma_{th}^{(ps)}$, a threshold stress for DHC initiation at planar surface



Threshold stress for DHC initiation is the lowest applied stress prior to hydride formation, at which DHC initiation may occur

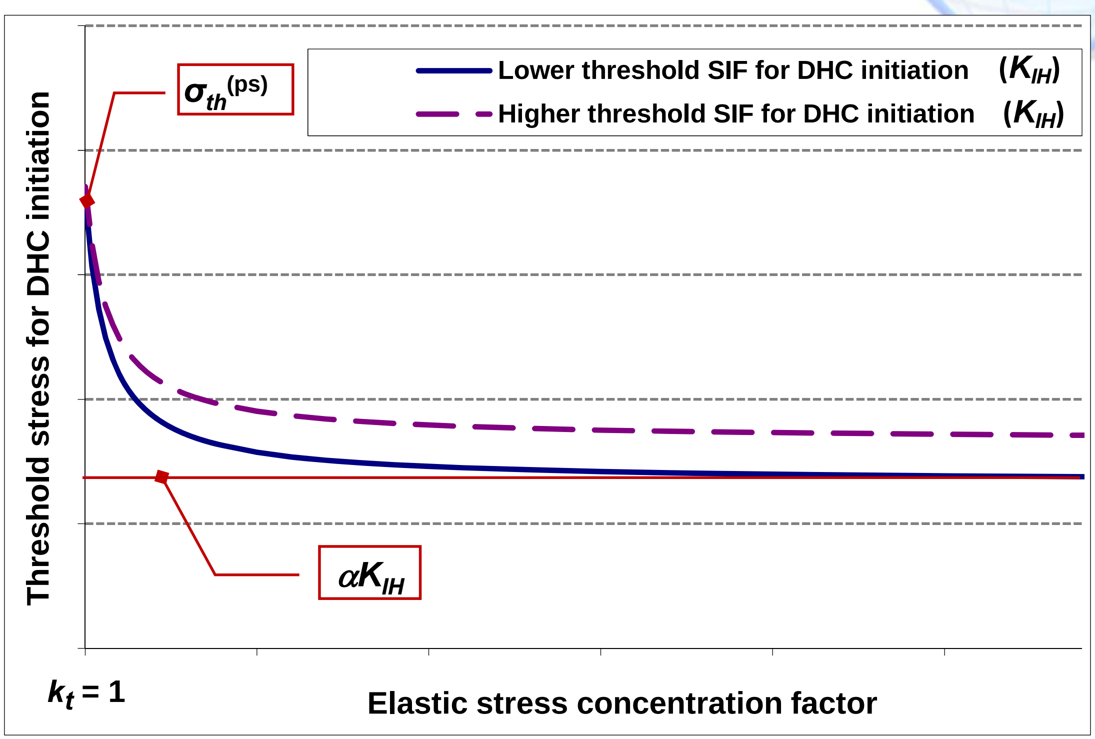
From: Scarth, D.A. and Smith, E., "Developments in Flaw Evaluation for CANDU Reactor Zr-Nb Pressure Tubes", Proceedings of ASME PVP Conference, Boston, MA, USA, 1999, PVP-Vol. 391, pp. 35-45.

Background

Threshold nominal stress for DHC initiation

$\sigma_{th,n}$ decreases monotonically as k_t increases

$\sigma_{th,n}$ increases monotonically as K_{IH} increases



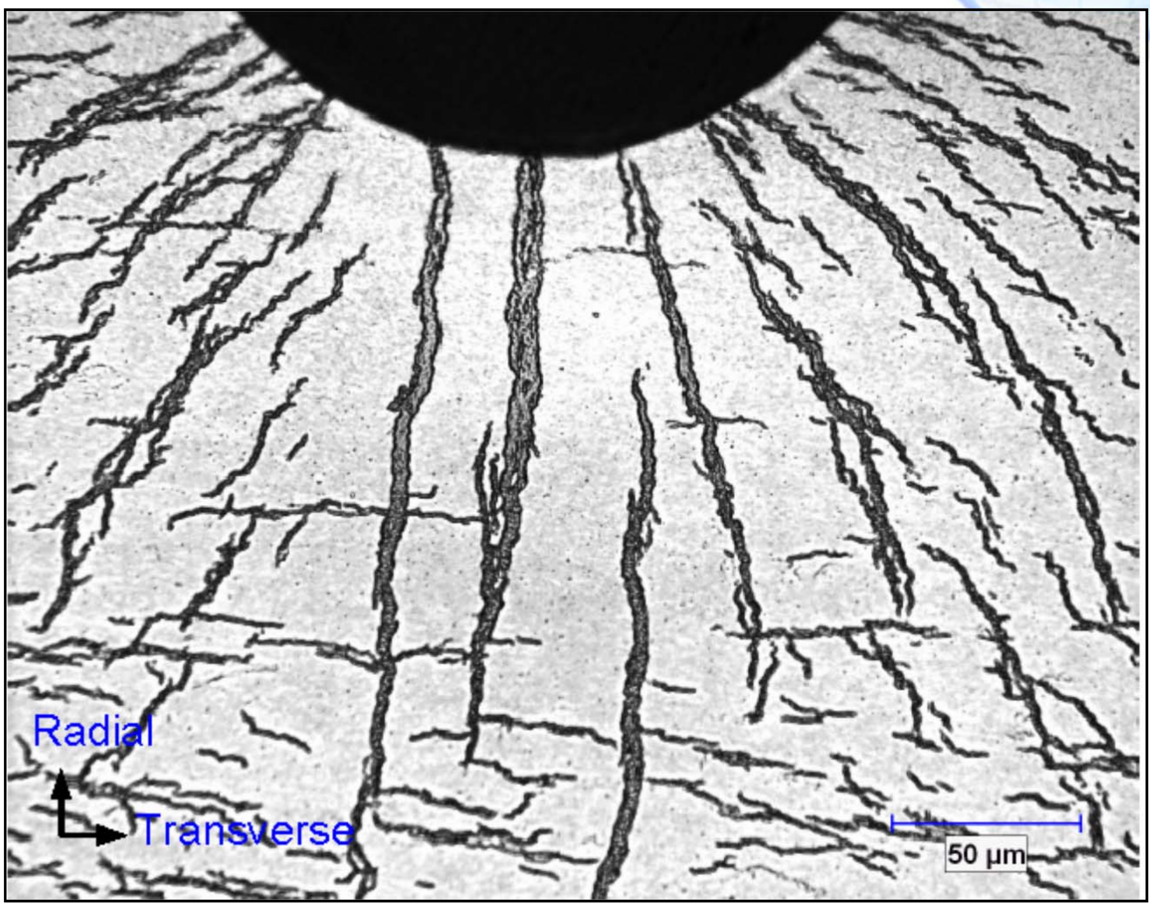
k_t : elastic stress concentration factor
K_{IH} : threshold stress intensity factor for DHC initiation from a crack

k_t = 1 for a planar surface ($\rho \to \infty$) and increases with decreasing ρ, approaching infinity for a crack ($\rho \to 0$)

Background

Hydrided regions in Zr-2.5Nb

example hydrided region at tip of V-notch in Zr-2.5Nb



From: Cui, J., Shek, G.K., Scarth, D.A. and Wang, Z., "Delayed Hydride Cracking Initiation at Notches in Zr-2.5Nb Alloys", Journal of Pressure Vessel Technology, August 2009, Vol. 131.