SENSITIVITY AND UNCERTAINTY ANALYSES FOR PROBABILISTIC LBB ANALYSES: METHODS AND APPLICATIONS

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Outline

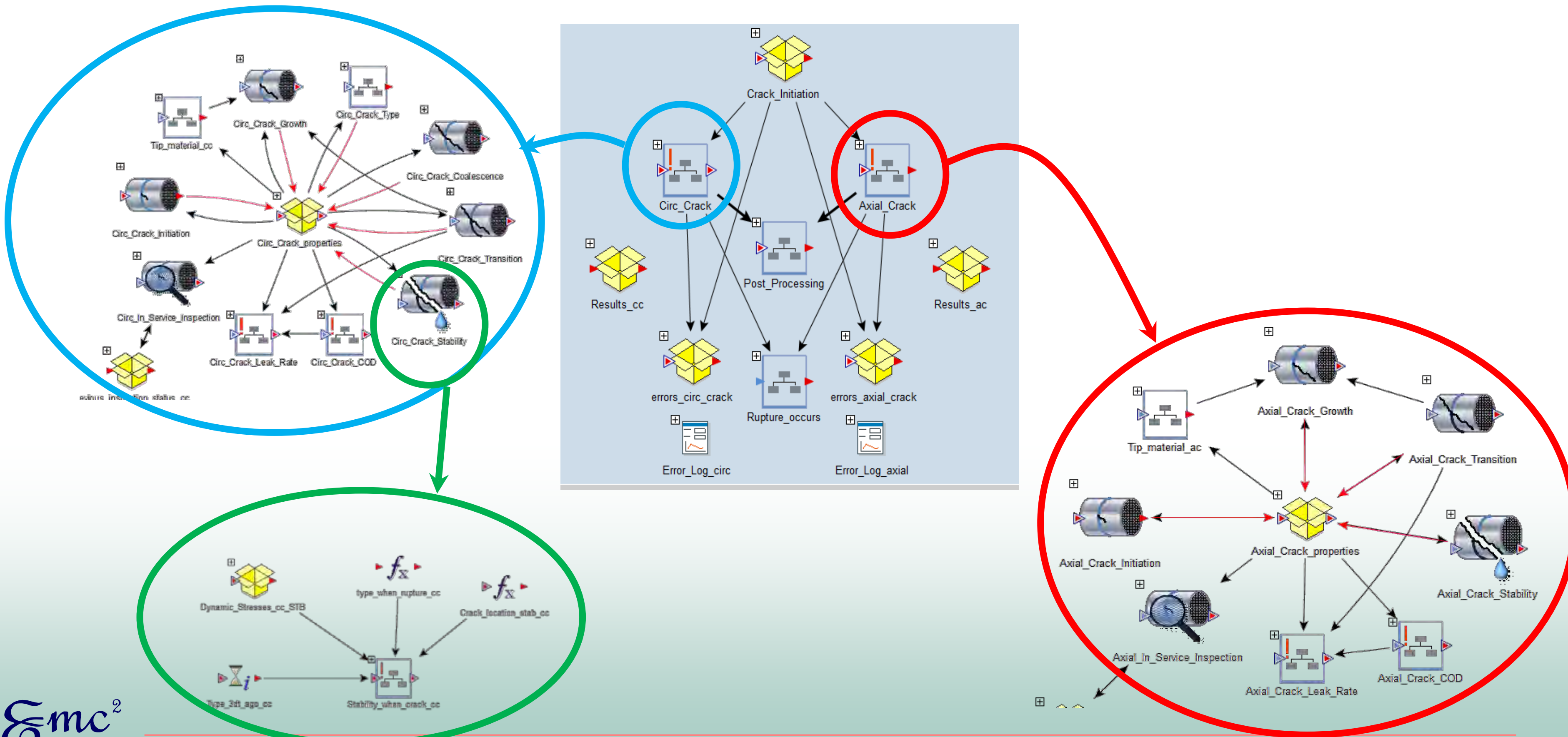
- xLPR presentation
- Uncertainty Characterization
- Uncertainty Propagation
- Output uncertainty and sensitivity analyses
- Conclusion

xLPR Presentation

- **xLPR :** Extremely Low Probability of Rupture
- Conjoint development between US-NRC and EPRI
- Assesses probability of rupture in nuclear piping systems but not limited to this particular outcome: a large number of other outputs are considered (first crack occurrence, first leakage, leak rate, increase in leaking ...)
- Develop as modular and flexible with large number of options:
  - **Two cracks orientation** : Axial and circumferential
  - **Two source of cracking** : PWSCC and Fatigue
  - **Many models** : Initiation, Growth, Coalescence, Transition, Stability, COD, leak rate…
  - **Several alternative options**: Mechanical (MSIP, Overlay, Inlay) and chemical (Zinc addition, Hydrogen concentration) mitigations, In Service Inspection, Leak Rate detection,…

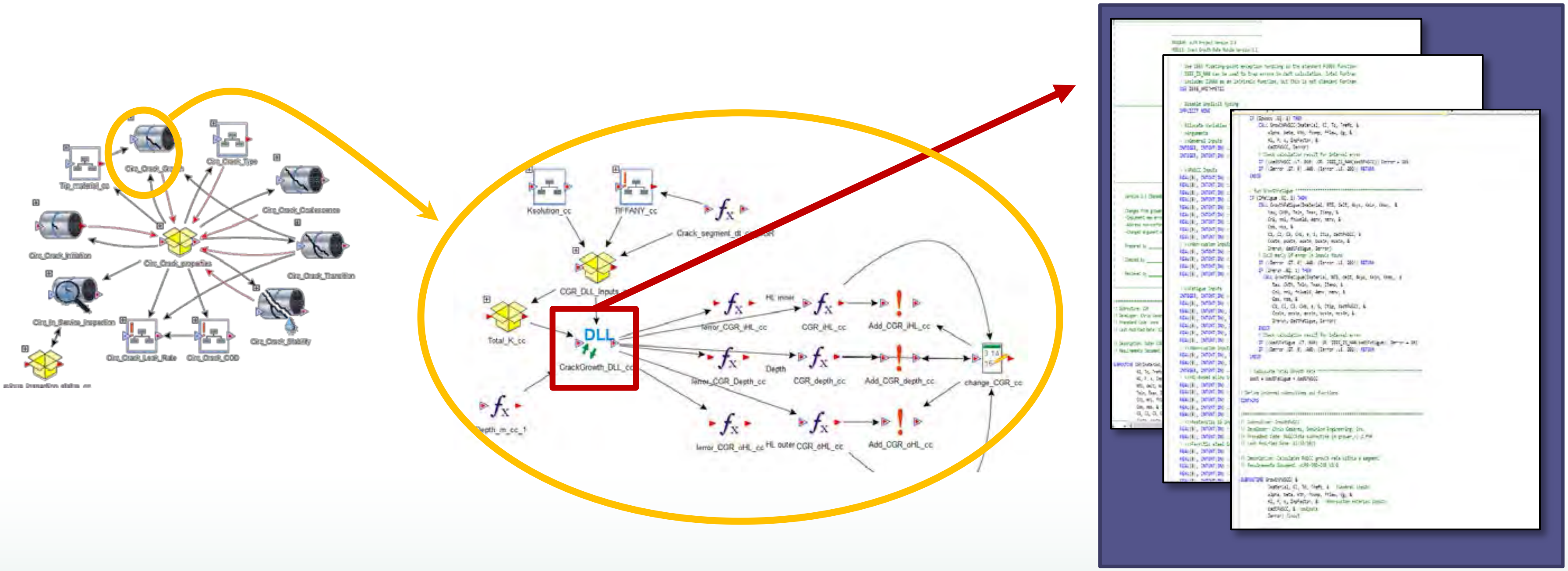
xLPR v2.0 code (1/2) – large and complex code

- Multi layer code with many interactions



xLPR v2.0 code (2/2) – large and complex code

- **Each DLL element in the code correspond to a module with many lines of code with specific physical concepts and assumptions**



- **This complexity translates to the user in the input workbook:**
  - More than 600 module input, parameters, scenarios options
  - Most of them can be **uncertain**

Uncertainty Characterization

- **Model uncertainty is captured via model parameters or options to use different models (initiation)**
- **Input and parameters uncertainty represented within a probabilistic framework.**
- **User can characterize each input and model's nature (constant or uncertain with distinction between aleatory and epistemic) then associate a nominal value as well as a distribution**

	Pre-Mitigation (5101-5199)									
5101	Log reg intercept param, beta_0 (circ)		Aleatory	no	0.5	5.41	NORMAL	5.41	3.64	
5102	Log reg slope param, beta_1 (circ)		Aleatory	no	0.5	0.86	NORMAL	0.86	6.02	
5103	Log reg intercept param, beta_0 (axial)		Aleatory	no	0.5	2.5	NORMAL	2.5	0.51	
5104	Log reg slope param, beta_1 (axial)		Aleatory	no	0.5	0.82	NORMAL	0.82	1.4	
5105	Depth-sizing bias term, a (circ)		Aleatory	no	0.5	0.018	NORMAL	0.018	0.017	
5106	Depth-sizing slope term, b (circ)		Aleatory	no	0.5	0.971	NORMAL	0.971	0.029	
5107	Depth-sizing bias term, a (axial)		Aleatory	no	0.5	0.007	NORMAL	0.007	0.011	
5108	Depth-sizing slope term, b (axial)		Aleatory	no	0.5	0.984	NORMAL	0.984	0.028	
5109	Sigma_depth (circ.)		Constant	no	0.5	0.04	DISCRETE	1	1	0.04
5110	Sigma_depth (axial)		Constant	no	0.5	0.066	DISCRETE	1	1	0.066
5111	Small flaw threshold, x_Small,Eval		Constant	no	0.5	0.1	DISCRETE	1	1	0.1
5112	Depth lower bound, x_LB		Constant	no	0.5	0.1	DISCRETE	1	1	0.1
5113	Depth repair threshold, x_TH		Constant	no	0.5	0	DISCRETE	1	1	0
	During and Post-Mitigation (5201-5299)									
5201	Depth repair threshold, x_TH (during)		Constant	no	0.5	0	DISCRETE	1	1	0
5202	Depth repair threshold, x_TH (post)		Constant	no	0.5	0	DISCRETE	1	1	0

Input distributions / Correlation

- **Distributions based on information gathered by specifically tasked group (Input Group):**
  - Distribution fitting using collected data when large number of data available
  - Distribution fitting and expert elicitation when small number (<10) data available
  - Expert elicitation when no data available.
- **Realistic input set requires correlation control when inputs varies accordingly (e.g., Yield Strength and Ultimate Strength of selected material)**
  - Due to size of problem (more than 500 inputs) hard wired correlations (user can select correlation value)
  - Rank correlation implemented. Treated between two selected variables and not as a whole large matrix

CORRELATIONS

ID	name	ID	name	location	(rank) correlation
2594	Peak-to-Valley ECP Ratio, P-1	2595	Charact Width of Peak vs ECP, c	PWSCC growth properties	0.714
2551	Weibull Vertical Intrcpt Error, EpsC	2552	General Weibull Slope, Beta	PWSCC initiation	-0.905
2525	Strain Threshold, STH	2528	Co	Fatigue Initiation	1
2592	Comp-to-Comp Variab Factor, fcomp	2543	Multiplier proport. Const. A (DM1)	PWSCC growth/init	0
2592	Comp-to-Comp Variab Factor, fcomp	2547	Multiplier proport. Const B (DM2)		0
2592	Comp-to-Comp Variab Factor, fcomp	2551	Weibull Vertical Intrcpt Error, EpsC		0
2501	Yield Strength, Sigy	2502	Ultimate Strength, Sigu	General Properties	0.709
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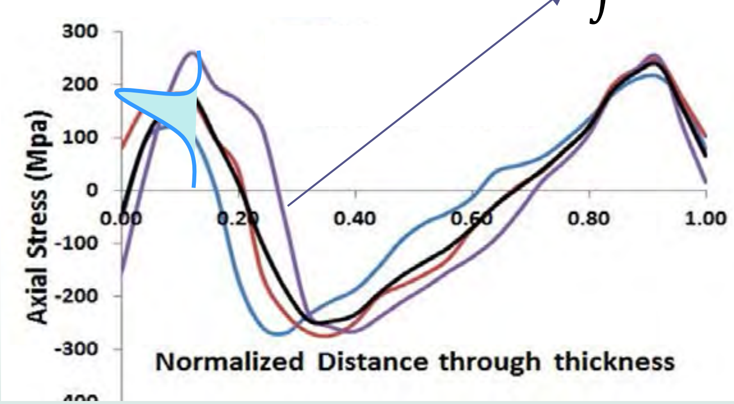
2506	Material Init J-Resistance, Jic	2507	Material Init J-Resist Coef, C		0

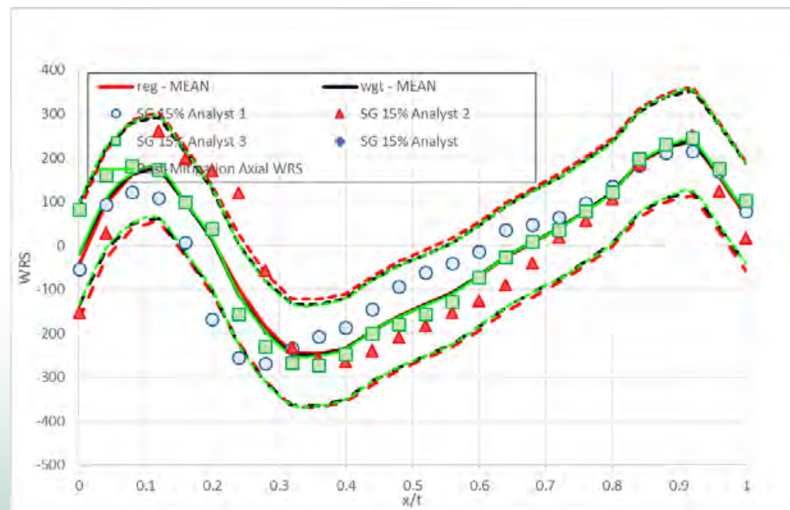
More complex uncertainty representation - WRS

- Uncertainty through the thickness
- Smoothness of curve
- Integral over thickness = 0 for axial WRS (equilibrium condition)

- Average standard deviation
- “adapted” correlation
- Construction of multiple profile and selection of profile closest to equilibrium

$$\int . = 0$$





More complex uncertainty representation - WRS

- **WRS representation is directed by the following constraints:**
  - Linear interpolation between 26 values through the thickness so that Universal Weight Function can be used
  - Uncertainty represented at each of the 26 locations
  - Uncertainty between the location needs to be correlated to avoid a sawtooth pattern
  - For axial WRS (used for circumferential cracks initiation and growth) axisymmetric conditions requires equilibrium (WRS over thickness integrates to zero).
- **Approach used to represent uncertainty conditional on all the above requirements:**
  - Average standard deviation through the thickness
  - Representation of uncertainty with normal distributions only
  - Estimate of correlation between point globally
  - Varying correlation value based on mean value for axial WRS
  - Sampling of 100 different profiles and select the one with integral closest to zero for axial WRS

Uncertainty Propagation

- Large number of uncertain inputs: more than 500 with some also spatially varying
- Several outputs of interest: probability of first crack occurrence over time and first leakage (for circ. crack and axial crack separately and combined), probability of pipe rupture, of Loss Of Coolant Accident (LOCA) …
- Optimization and Derivation based methods would be *initially* either too costly due to the size of the problem or would require to make some assumption
- Sampling-based methods (aka Monte Carlo methods) preferred method of uncertainty propagation
- Inputs are sampled by sampling from uniform distribution and use inverse CDF for each input distribution

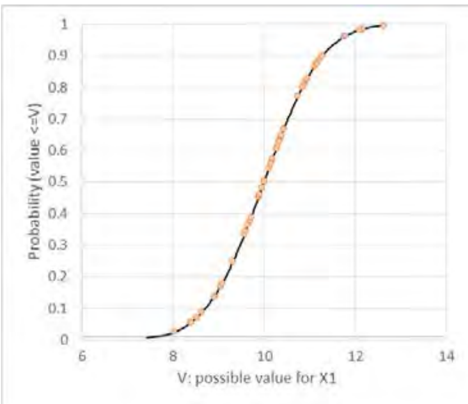
Monte Carlo strategies

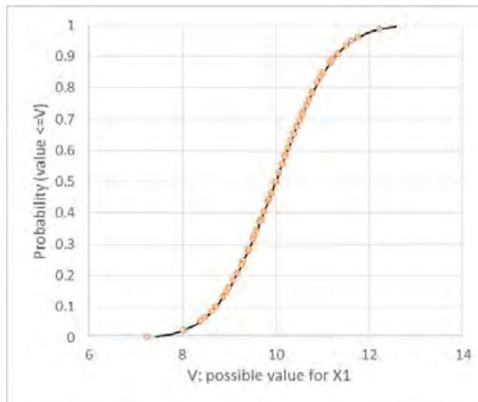
Three Monte Carlo techniques are available in xLPR

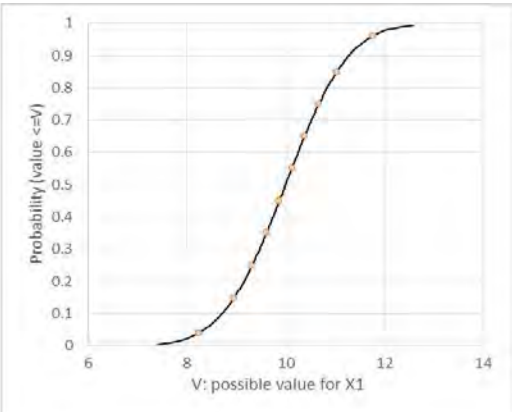
- **Simple Random Sampling (SRS)** : regular Monte Carlo technique – sample size can be expanded easily if convergence not reached
- **Latin Hypercube Sampling (LHS)**: with dense stratification of each input for better mono-dimensional coverage – variance in the estimate usually reduced compared to SRS
- **Discrete Probability Distribution (DPD)**: with discretization of input reducing mono-dimensional coverage for better multi-dimensional coverage.

- **All techniques compatible with**
  - Correlation control
  - Separation epistemic/aleatory
  - Use of importance Sampling
  - Representation with many possible distribution types

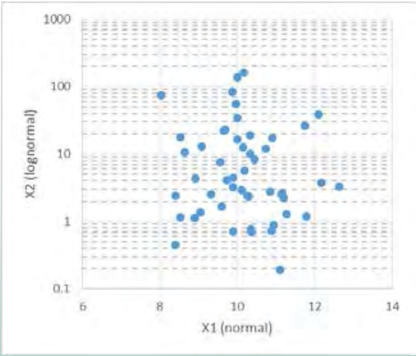
- Monodimensional coverage (left – SRS ; Middle LHS ; right DPD)

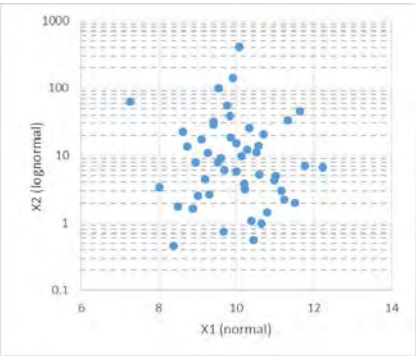


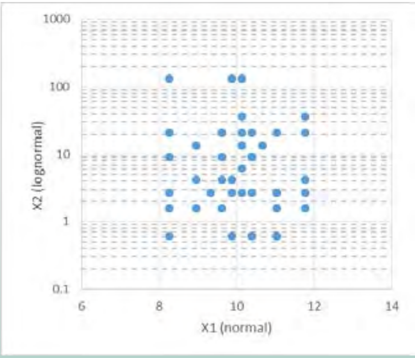




- Multi dimensional coverage (left SRS ; middle LHS ; right DPD)

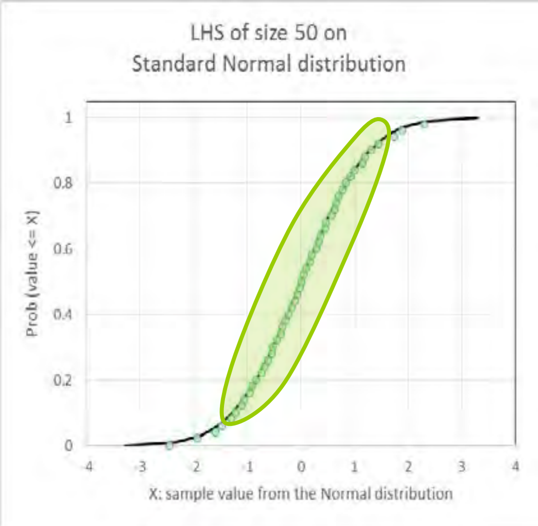




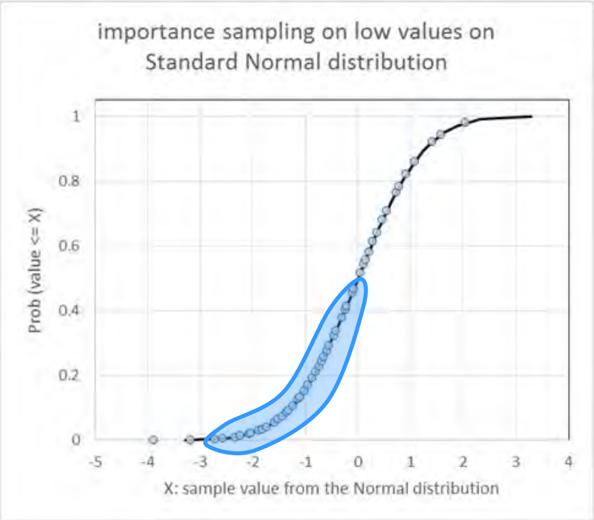


Importance Sampling

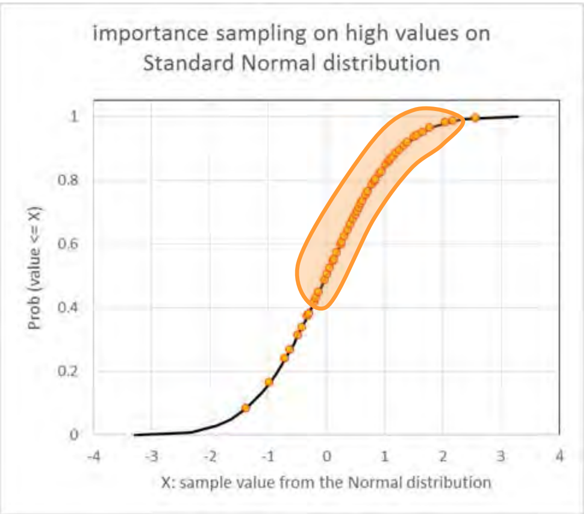
- Importance Sampling allows to focus in the region of the input space of greater importance (e.g., leading to more rupture).
- Works by "biasing" the selecting distribution (increasing the density) around an area of interest to sample more value in this region
- Each realization is weighted to reflect the initial distribution
- Requires to know which inputs and which values for these inputs are important



Initial Sample



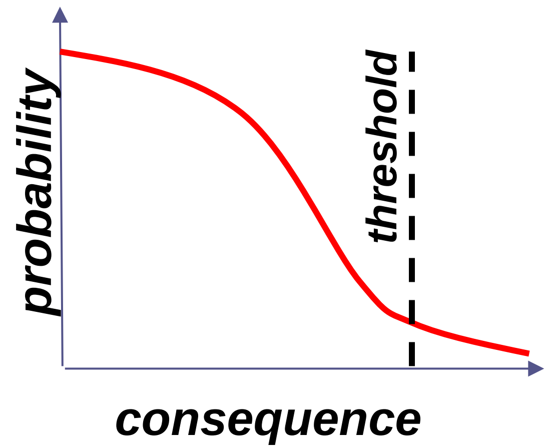
Importance on low value

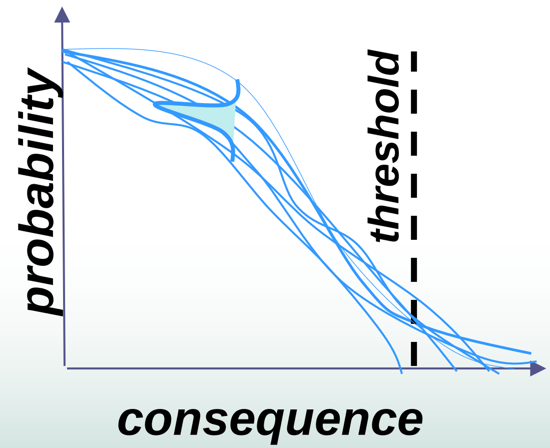


Importance on high values

Epistemic and Aleatory uncertainty

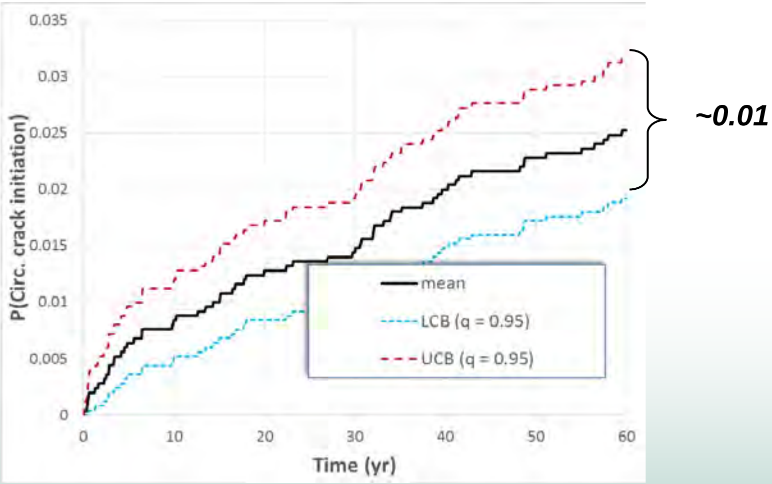
- Aleatory uncertainty represent randomness of (usually future) events. Cannot be reduced. Represents the risk of having adverse condition
- Epistemic uncertainty represents the lack of knowledge toward a value supposed to be fixed but poorly understood. It usually can be reduced thanks to more research, experiment or gathering more data. It represent the uncertainty over the risk
- Several risk analyses keep the distinction between aleatory and epistemic uncertainty to gain more insights in the system under consideration.
- When Monte-Carlo sampling techniques are used, the separation is generated via a nested loop with the outer loop used to sample epistemic uncertainty and inner loop the aleatory uncertainty

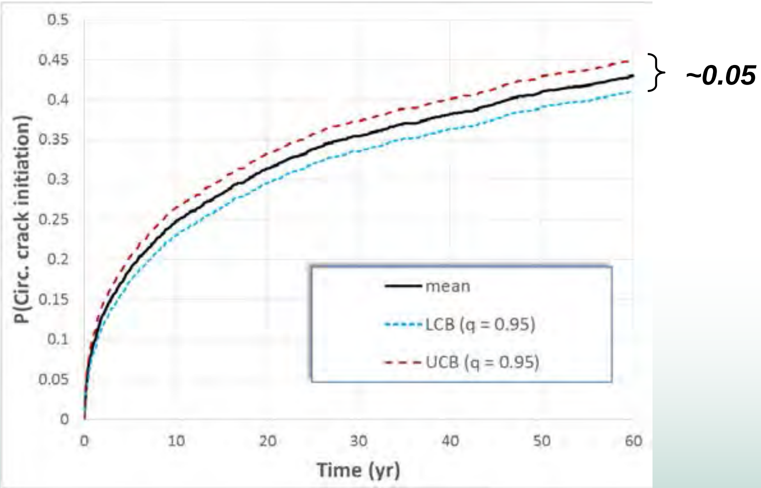




Output Uncertainty Analysis

- The purpose of uncertainty analysis is to summarize output of interest in a comprehensive way that helps and supports any decision making
- Indicator are both quantitative (comparison with respect to a threshold value, estimate of uncertainty) and qualitative (graphical representation)



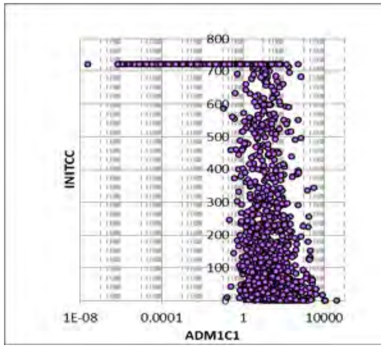


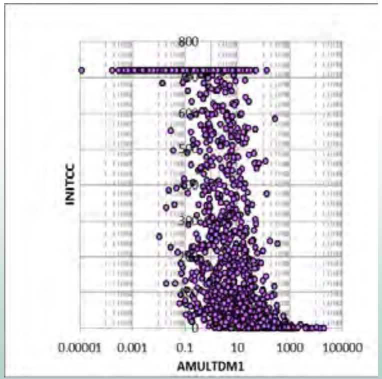
Distinction between relative difference vs. absolute difference

Output Sensitivity Analysis – traditional approach

- Purpose: understand how the input uncertainty affects the output uncertainty and rank inputs by importance
- Traditional approach include Stepwise (rank) regression : construction of linear/monotonic model and scatterplots
- Works 60% to 80% of the time as most of the input influence is monotonic
- However, mitigation, human interaction may create non-monotonic influence (obvious cases detected)
- And it is common to have conjoint influence
- In this case stepwise regression (monotonic and additive) is not sufficient.

	Rank Regression		
Final R^2	0.83		
Input	R^2 inc.	R^2 cont.	SRRC
ADM1C1	0.66	0.66	-0.52
AMULTDM1	0.80	0.14	-0.34
ADM1C2	0.82	0.01	-0.10
AXIALWRS	0.82	0.01	-0.07
ADM1C3	0.83	0.01	-0.06
FFLAWA5	0.83	0.00	0.01
ADM1C4	0.83	0.00	-0.03
AMD1A2	0.83	0.00	-0.03
HOOPWRS	0.83	0.00	0.01
ADM1A3	0.83	0.00	-0.02
EFPY	0.83	0.00	-0.01
ADM1C5	0.83	0.00	-0.01
ADM1A1	—	—	—
COA5	—	—	—
FFLAWC4	—	—	—
STHA5	—	—	—
STHMULT	—	—	—





Output Sensitivity Analysis - ANOVA

- **Other regressions can capture non-monotonic and conjoint influence. However they do not give directly ranking of inputs with respect to their uncertainty's influence.**
- **Two step method approach**
  - Creation of surrogate model (response surface) based on available data
  - Estimation of the response surface quality via coefficient of determination (R^2)
  - Using analytical response surface to estimate importance of the input via Sobol decomposition for the Analysis of Variance (ANOVA)
- **Two non-monotonic regressions considered: Recursive partitioning (tree-based) and Multi-adaptive splines (MARS)**
- **Influence of parameter estimated using three regressions (stepwise regression is added)**
- **Regression analysis used for understanding important parameters and not for prediction!**

Conclusion

- Probabilistic Fracture Mechanics is an important part of any risk analysis or risk-informed approach
- US NRC and EPRI worked conjointly to develop xLPR v2.0 in order to perform probabilistic LBB analyses
- The code is developed to be modular and includes many options to extend its use to several problems
- Uncertainty represented via probabilistic framework (and probability distributions)
- Uncertainty is propagated using sampling based methods with variety of combination available
- Uncertainty and sensitivity analyses techniques are used to analyses the output and support decision making process.